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## **Hollow circular-truncated cone resonator and its hollow variable biconical laser beam**

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# Hollow circular-truncated cone resonator and its hollow variable biconical laser beam

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**Abstract.** To obtain a hollow variable biconical laser beam (HVBLB), a CO<sub>2</sub> laser having a hollow circular-truncated cone resonator (HCTCR) is presented. This HCTCR comprises a rotationally symmetric total-reflecting concave mirror at the bottom, a rotationally symmetric part-reflecting convex mirror at the top, and a hollow circular-truncated cone discharge tube at the middle. The cross section of this generated biconical laser beam changes from annulus to circular to annulus and the size of this cross section from big to small to large as the propagation distance increases. So, a kind of laser beam with variable center intensity from zero to peak value to zero is obtained and is known as HVBLB. Due to the inclusion of part of the hollow laser beam (HLB) and solid laser beam, this HVBLB requires no additional beam-shaping element and has broad applications such as optical trapping and commercial manufacturing. © The Authors. Published by SPIE under a Creative Commons Attribution 3.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: 10.1117/1.OE.53.5.056113]

Keywords: hollow circular-truncated cone resonator; hollow variable biconical laser beam; CO<sub>2</sub> laser; hollow laser beam.

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## 1 Introduction

Hollow laser beam (HLB) means there is a dark region of intensity in the central axis of a beam. Due to the specific characteristic of intensity distribution of this beam and its potential application prospects, methods for obtaining HLBs have been extensively reported such as in Refs. 1–10. Among these methods, an HLB obtained directly by a hollow cone-shaped resonator (HCR) CO<sub>2</sub> laser has been proposed according to Ref. 7. As minimum spot of the output beam is located in its surface, the output mirror of this HCR should support higher power density. Also, the dark region of this HLB will become bigger as the transmission distance increases. So, this HLB must depend on a focusing system for various applications.

In this article, to improve power density of the output mirror and raise the distance between the smaller dark region and the output mirror, we present a new CO<sub>2</sub> laser, whose resonator has a hollow circular-truncated cone, shown in Fig. 1. We will show that the hollow situation of output beam of this resonator changes as the propagation distance increases. Also, the output mirror of the hollow circular-truncated cone resonator (HCTCR) will face lower power density due to the minimum spot lying outside the output mirror.

## 2 HCTCR and Its Hollow Variable Biconical Output Beams

The HCTCR CO<sub>2</sub> laser includes a rotationally symmetric concave total reflector at the bottom, a rotationally symmetric convex part reflector at the top, and a hollow circular-truncated cone discharge tube at the middle, as shown in Fig. 1. Also, laser beams emit from R<sub>2</sub> in Fig. 1(a). Figure 1(d) is a section of the HCTCR along the axis of symmetry. In Fig. 1(d), R<sub>11</sub> is concave total reflector, R<sub>21</sub> is

convex part reflector, and both R<sub>11</sub> and R<sub>21</sub> form a concave-convex sub-branch cavity of the HCTCR. This sub-branch cavity and another sub-branch cavity formed of R<sub>12</sub> and R<sub>22</sub> are symmetric about the z-axis. So, simply put, the HCTCR is formed by a R<sub>11</sub>–R<sub>21</sub> concave-convex sub-branch cavity making a circuit of the z-axis. So, the output beams from the HCTCR are also formed by the output beams of R<sub>11</sub>–R<sub>21</sub> cavity making a circuit of the z-axis. In Ref. 7, there was a similar theory.

The main difference between the HCTCR and the HCR is the concave-convex sub-branch cavity in this case instead of concave-plane-concave folded sub-branch cavity in Ref. 7. This reflects the fact that the location of minimum spot can be adjusted to locate outside this resonator. So, the output beams of the HCTCR can have a more abundant intensity distribution.

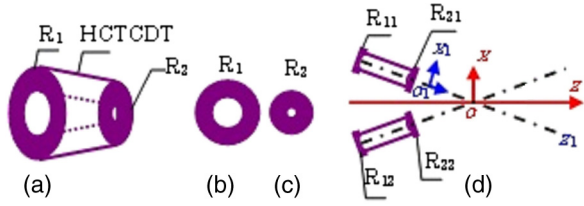
To obtain the suitable minimum spot size, the location of minimum spot, and the half-apex angle, we can regulate and control the values of R<sub>11</sub>, R<sub>21</sub>, the distance between R<sub>11</sub> and R<sub>21</sub>, and the angle between the optical axis of R<sub>11</sub>–R<sub>21</sub> cavity and the z-axis, according to the resonator theory. As we are interested in the behavior of the output beams of the HCTCR here, this hypothesis is feasible.

According to Ref. 7, assuming the output beam from the R<sub>11</sub>–R<sub>21</sub> sub-branch cavity is a fundamental Gaussian beam and linearly polarized along the x<sub>1</sub>-axis. The field distribution  $E(x_1, 0, xz_1)$  in coordinates  $(x_1, 0, z_1)$  can be defined by

$$E(x_1, 0, z_1) \propto C_0 \frac{\omega_{0x_1}}{\omega_{x_1}(z_1)} \exp \left\{ -i \left[ kz_1 - \arctan \left( \frac{\lambda z_1}{\pi \omega_{0x_1}^2 n} \right) \right] \right\} \times \exp \left\{ -x_1^2 \left[ \frac{1}{\omega_{x_1}^2(z_1)} + \frac{ik}{2R_{x_1}(z_1)} \right] \right\}, \quad (1)$$

where  $C_0$  is a constant,  $\omega_{0x_1}$  is minimum spot size in plane  $(x_1, 0, z_1)$ ,  $\omega_{x_1}(z_1)$  is spot size in the  $z_1$  place of plane  $(x_1, 0, z_1)$  and is equal to  $\omega_{0x_1} [1 + (\lambda z_1 / \pi \omega_{0x_1}^2 n)^2]^{1/2}$ ,

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**Fig. 1** (a) Skeleton map of hollow circular-truncated cone resonator (HCTCR). (b) Elevation view of annular total reflector. (c) Elevation view of annular partial reflector. (d) Section of HCTCR. HCTCRD is hollow circular-truncated cone discharge tube.  $R_1$  is annular total reflector.  $R_2$  is annular partial reflector.  $R_{11}$  and  $R_{12}$  are sections of annular total reflector.  $R_{21}$  and  $R_{22}$  are sections of annular partial reflector.  $R_{11} - R_{21}$  cavity and  $R_{12} - R_{22}$  cavity are the sub-branch cavity of the HCTCR, and they are symmetric about the  $z$ -axis. The HCTCR is formed by  $R_{11} - R_{21}$  cavity (or  $R_{12} - R_{22}$  cavity) making a circuit of the  $z$ -axis. Also, the output beams from the HCTCR are formed by the output beams of  $R_{11} - R_{21}$  cavity (or  $R_{12} - R_{22}$  cavity) making a circuit of the  $z$ -axis.

$R_{x1}(z_1)$  is the radius of curvature and equal to  $z_1[1 + (\pi\omega_{0x1}^2 n/\lambda z_1)^2]^{1/2}$ ,  $k = 2\pi n/\lambda$  is the wave number, and  $n$  is the index of refraction.

The relationship between  $(x_1, 0, z_1)$  coordinates and  $(x, 0, z)$  coordinates can be expressed by

$$\begin{cases} x_1 = x \cos \theta + z \sin \theta \\ z_1 = z \cos \theta - x \sin \theta + d \end{cases} \quad (2)$$

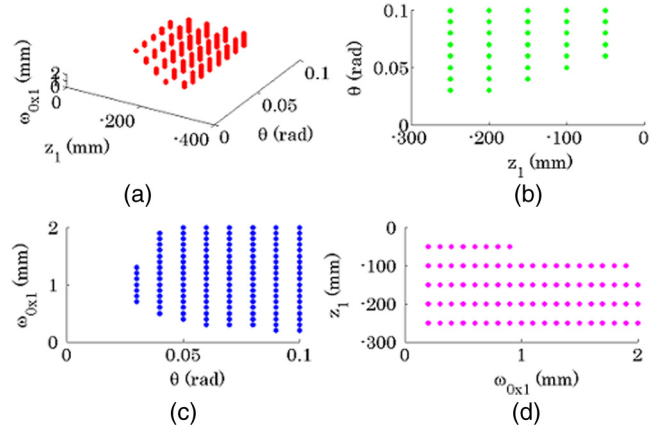
where  $d$  is the distance between the point  $o_1$  and  $o$ . Sign of  $d$  is positive if the point  $o_1$  is located in between  $R_{21}$  and the point  $o$ ,  $d$  is negative if the point  $o$  is located in between  $R_{21}$  and the point  $o_1$ , and  $d$  is equal to zero if the point  $o$  and  $o_1$  have the same position. Also,  $\theta$  is the half-apex angle of the HCTCR.

Substituting Eq. (2) in Eq. (1), we can obtain the field distribution  $E(x, 0, z)$  from the  $R_{11} - R_{21}$  sub-branch cavity in coordinates  $(x, 0, z)$ . Using the  $E(x, 0, z)$  to rotate a circle around the  $z$ -axis, the field distribution  $E(x, y, z)$  of the output beams from the HCTCR can be obtained. Also, the intensity distribution of the output beams of the HCTCR can be described by  $I \propto E(x, y, z)E^*(x, y, z)$ . Here, we can imagine that the output beam from this HCTCR will change from hollow center to solid center to hollow center as the propagation distance increases. Therefore, this output beam is named hollow variable biconical laser beam (HVBLB).

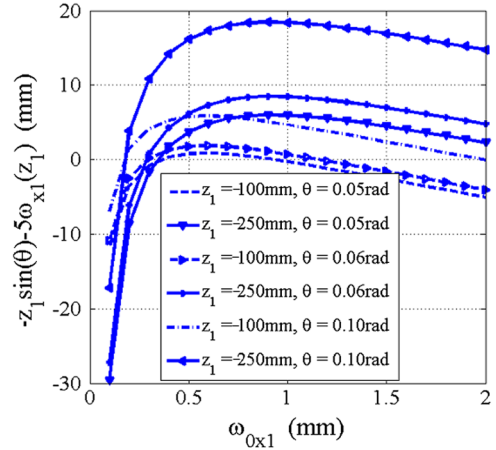
### 3 Simulation and Discussion

The parameters used in the calculation are wavelength  $\lambda = 10.6 \mu\text{m}$ , the refractive index  $n$  in free space is 1, and the constant  $C_0$  is 1. As for other parameters, some factors need to be considered. First, the radius of every mirror in the sub-branch is based on the spot size on this mirror. By the definition of spot size, it is best to have the former size above  $\sqrt{2}$  times the latter case. Second, from the inner edge of every mirror to the  $z$ -axis, the distance must be enough for the electrode and the holder. Third, the distance between the point  $o_1$  and  $o$  will be selected according to actual requirement.

According to the above-mentioned first and second principles, for the position of the mirrors,  $|d - z_1| \sin(\theta) > 5\omega_{x1}(z_1)$  may be the appropriate choice. Figures 2 and 3 take the relations among  $\omega_{0x1}$ ,  $z_1$ ,  $\theta$ , and  $|d - z_1| \sin(\theta) - 5\omega_{x1}(z_1)$ . Here,  $z_1$  is abscissa of mirrors in coordinates



**Fig. 2** If  $|d - z_1| \sin(\theta) - 5\omega_{x1}(z_1) > 0$ , (a) is the relationship among  $\omega_{0x1}$ ,  $z_1$ , and  $\theta$ , (b) is the relationship between  $z_1$  and  $\theta$ , (c) is the relationship between  $\omega_{0x1}$  and  $\theta$ , and (d) is the relationship between  $\omega_{0x1}$  and  $z_1$ . When  $d$  is equal to zero,  $z_1$  is the place of the output mirror,  $\omega_{x1}(z_1)$  is the spot size of the place  $z_1$ ,  $\omega_{0x1}$  is the minimum spot size of the sub-branch cavity, and  $\theta$  is the angle between the  $z_1$  and  $z$  axis.

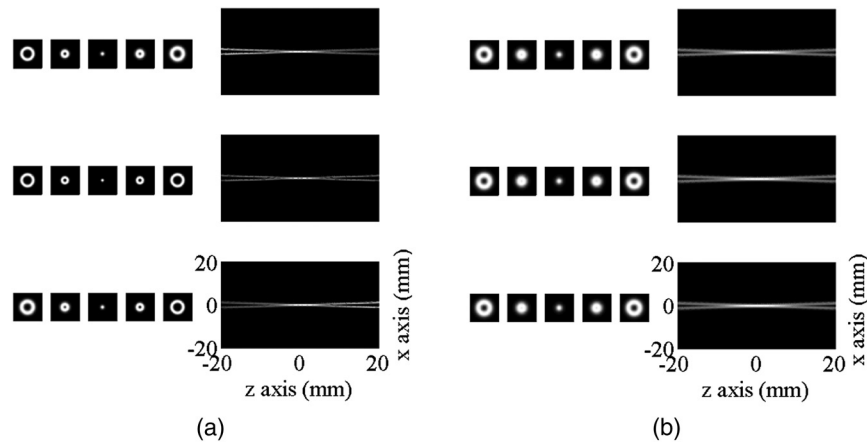


**Fig. 3** Variation of  $-z_1 \sin(\theta) - 5\omega_{x1}(z_1)$  according to  $\omega_{0x1}$  while  $z_1 = -100 \text{ mm}$ ,  $-250 \text{ mm}$ , respectively, and  $\theta = 0.05, 0.06$ , and  $0.10 \text{ rad}$ , respectively. Here,  $d = 0$  and  $|d - z_1| \sin(\theta) - 5\omega_{x1}(z_1) = -z_1 \sin(\theta) - 5\omega_{x1}(z_1)$ , because  $z_1$  is negative.

$(x_1, 0, z_1)$ , the sign of  $z_1$  here is negative due to two mirrors on the left of the point  $o_1$ .

From Fig. 2, if  $|d - z_1| \sin(\theta) - 5\omega_{x1}(z_1) > 0$ , the relationship among the  $\omega_{0x1}$ ,  $z_1$ , and  $\theta$  are shown in part (a), between  $z_1$  and  $\theta$  in part (b), between  $\theta$  and  $\omega_{0x1}$  in part (c), and  $\omega_{0x1}$  and  $\theta$  in part (d). Based on the data of Fig. 2, Fig. 3 shows some specific examples for ease of choice of parameters. For simplification,  $d$  is equal to zero in these two figures. The various values about  $d$  will be considered carefully in the following discussions.

According to the above-mentioned theory, the HVBLBs from the HCTCR at the  $\theta = 0.06 \text{ rad}$  have been simulated in Fig. 4. In Figs. 4(a) and 4(b), the minimum spot size  $\omega_{0x1}$  is equal to  $0.4$  and  $1.5 \text{ mm}$ , respectively. Three rows in every part of Fig. 4 show  $d = 50, 0$ , and  $-50 \text{ mm}$ , respectively. The six pictures in every row of every part are the spot image at  $z = -20, -10, 0, 10$ , and  $20 \text{ mm}$  and the longitudinal section of transmission beams from  $z = -20$  to  $20 \text{ mm}$ , respectively.



**Fig. 4** The hollow variable biconical laser beam (HVBLB) while the minimum spot size  $\omega_{0 \times 1} = 0.4$  mm in part (a) and  $\omega_{0 \times 1} = 1.5$  mm in part (b). Both part (a) and part (b) have 3 rows (i.e.,  $i = 1, 2, 3$ ) and 6 columns (i.e.,  $j = 1, 2, 3, 4, 5, 6$ ) although  $i = 1, 2, 3$ ,  $d = 50, 0$ , and  $-50$  mm, respectively. Although  $j = 1, 2, 3, 4, 5$ ,  $z = -20, -10, 0, 10$ , and  $20$  mm, respectively,  $j = 6$  is the longitudinal profile map of propagation of HVBLB along  $z$ -axis.  $a_{ij, j \neq 6}$  and  $b_{ij, j \neq 6}$  are spot image at difference position in  $z$ -axis. For example,  $a_{23}$  is the spot image and is located in the second row and third column of part (a), while  $\omega_{0 \times 1} = 0.4$  mm,  $d = 0$  mm, and  $z = 0$  mm, and  $b_{36}$  is the longitudinal profile map of propagation of HVBLB along  $z$ -axis and is located in the third row and sixth column of part (b), while  $\omega_{0 \times 1} = 1.5$  mm and  $d = -50$  mm. Here, the wavelength is  $10.6 \mu\text{m}$  and  $\theta$  is  $0.06$  rad.

Figure 4 shows that the output beams from the HCTCR have changed from hollow beams to center-bright beams and hollow beams with the increase in transmission distance. Also, the first hollow beams are named the left hollow beams, and the second hollow beams are named the right one. By comparing parts (a) and (b) of Fig. 4, it is clear that the length and width of the center-bright beams are influenced by the minimum spot size  $\omega_{0 \times 1}$  and the half-apex angle  $\theta$ . The smaller the minimum spot size and the greater the half-apex angle, the smaller the length and width of the center-bright beams. From three row pictures of every part, the hollow situation depends on the relative position and distance between the point  $o_1$  and  $o$ . If the point  $o_1$  is located to the right of the point  $o$ , the right hollow beams have the clearer bright ring and relatively thinner wall than the left one. So, if it is needed to use the right hollow beam, the point  $o_1$  would be set to the right of the point  $o$ .

According to the theory of cavity algebra of lasers, the radii of curvature  $R_{11}$  and  $R_{21}$  can be calculated by  $\omega_{0 \times 1}$ ,  $z_1$ , and length of sub-branch cavity.

From Fig. 4, the HVBLB would be influenced by the values of  $\omega_{0 \times 1}$ ,  $d$ , and  $\theta$ . In optical trapping, for smaller dark center of beam, the values of  $\omega_{0 \times 1}$  and  $\theta$  should be as small as possible and the value of  $d$  can be equal to zero. For large distances and high-power laser, bigger  $\omega_{0 \times 1}$  and smaller  $\theta$  should be chosen. If making a hole in short distance, proper  $\omega_{0 \times 1}$ ,  $d$ , and  $\theta$  need to be designed according to the size of the hole and the material of the object.

#### 4 Conclusions

Based on the design in this article, the position of the highest power density of the output beams is located outside of the HCTCR. The benefits have two advantages: First, the output mirror can be protected because of the smaller power density. This merit is especially useful for a high-power situation. Second, a variation beam from hollow to solid to hollow along the increase of the propagation distance can be obtained directly with this laser resonator. So, this output beam is

named as HVBLB. Both the hollow part and center-bright part of this HVBLB can be used directly without shaping and therefore do not waste extra energy. The output beams will be used for not only small-power application areas but also high-power cases and have extensive prospects.

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