COMMUNICATIONS

Comment

Comment on "Special section on wavelet transforms": Optical versus digital implementation of the wavelet transform

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A special section in the September 1992 issue of *Optical Engineering* is devoted to wavelet transforms. Upon studying several papers, a reader might be lead to believe that digital computations of the wavelet transform are slow and inaccurate. For example, in the introduction to the special section, Szu and Caulfield state that wavelet transforms are "computationally intense," "even 'fast' wavelet transforms can be quite slow digitally," and "a slight error in the digital computing of shift variables can produce a large error in wavelet coefficients." [1] In another paper, Caulfield and Szu provide an example to show that digital computations of the wavelet transform will tax the power of a supercomputer.[2]

However, it is well documented that digital computations of both the continuous and the discrete wavelet transform are extremely fast for wavelets based on recursive multirate digital filter bank structures.[3–7] The computational cost of the transform is $O(\log_2 L)$ multiplications and additions per output point where L is the length of the digital filter.[4] Furthermore, for N sample points, only $O(N)$ points in the wavelet transform domain are needed to compute perfectly (within numerical precision) the inverse wavelet transform. Thus, for 1-D signals the computation is $O(N \log_2 L)$, and for $N \times N$ 2-D signals it is $O(N^2 \log_2 L)$.

In most applications the support of the wavelet, which is a function of L, is selected to be much smaller than N to analyze the nonstationary or local properties of a signal.[8] Therefore, the computation of the wavelet transform is faster than the fast Fourier transform (FFT) by a factor of $\log_2 N / \log_2 L$. Indeed, it is the extremely fast computation speed that motivates much of the interest in the wavelet transform and its applications.

It is well known that the discrete wavelet coefficients are different for a signal and a shifted version of that signal.[9,10] This property in no way affects the *accuracy* of the transform. The transform coefficients of the original signal reconstruct the original signal, and the transform coefficients of the shifted signal reconstruct the shifted signal. Of course, the digital computation is not perfect due to filter coefficient quantization and roundoff effects, but in multirate filter banks these errors are generally very small.[11]

For comparison, it is useful to estimate the current potential computation speeds of optical and digital technologies. Several proposed optical wavelet systems in the September 1992 issue of *Optical Engineering* are based on optical correlators. Flannery and Horner state that optical correlators have "the potential to perform correlations of reasonably high resolution (e.g., 512×512 pixels) images at rates approaching 1000 frames per second. . . ." [12] We can, therefore, assume that wavelet systems based on optical correlators also have the potential of computing 512×512 wavelet transforms at a rate of 1000/s using several correlators in parallel.

Now consider the digital VLSI technology required to compute 512×512 FFTs at a rate of 1000/s. Using data from a FFT chipset developed by Honeywell, a 16-bit 512-sample complex FFT can be computed in 10.72 µs with five chipsets running in parallel (operating at 50 MHz, 2 chips per chipset, a total of 260K transistors for each chipset).[13] A single 512×512 complex FFT could then be computed[14] in $2 \times 512 \times 10.72 \times 10^{-6} = 11$ ms. And with 55 chipsets, or more conservatively, 64 chipsets ($64 \times 0.26\text{M} = 17\text{M}$ transistors), the FFTs could be computed at a rate of 1000/s.

Current integrated circuits[15,16] have integration levels of up to 3.1M transistors and clock rates of up to 200 MHz. With this technology, considerably fewer chips would be required to compute the FFTs. By combining four chipsets (1M transistors) onto a single chip operating at 66 MHz, the FFTs could conceivably be computed using 12 chips. (The Intel 486DX2 microprocessor[15] has 1.2M transistors and operates at 66 MHz.) Such a processing system could then be integrated into a small package by means of silicon multichip modules. Rockwell, for example, has developed a digital signal processing system capable of 400 million floating point operations per second by combining 12 Texas Instruments TMS320C30 (33 MHz) digital signal processors, associated memory, and interface logic into a 75-g 8.3- $\times$ 8.3- $\times$ 0.95-cm package.[17]

The hardware requirements of the FFT provide a means of estimating the hardware requirements of the wavelet transform. For example, with the choice $L = 8$, the wavelet transform requires roughly three times fewer computations than a 512×512 FFT. It is reasonable to conclude that high-accuracy 512×512 wavelet transforms at rates of 1000/s could be computed with a moderate number of custom integrated circuits combined into a compact package by means of current or near-term technology. The only real drawback of implementing the wavelet transform by means of 50 + MHz VLSI technology is its higher power consumption.[18] Whether such a digital system will ever be developed, of course, depends on whether there are enough economic incentives.

The preceding estimates of digital performance are for wavelets computed by means of multirate filter banks. It might be argued that the set of wavelets generated by a multirate filter bank structure is small and has only limited applicability, but considerable progress has been made in de-

signing these wavelets. Wavelets based on filter banks have been designed to match a signal of interest, to be nonseparable in multiple dimensions, and to have noninteger (rational) scaling factors.[19–21] Furthermore, fine-sampled (or oversampled) transforms in both the scale and the shift parameter can be efficiently computed.[4]

Considering the speed and accuracy with which wavelet transforms can be computed by means of digital systems, the practicality of optical systems that use these wavelets is questionable. As a case in point, two papers in the September 1992 issue of *Optical Engineering* discuss extremely low accuracy (1-bit) optical computations of the wavelet transform using the Haar wavelet.[22,23] But the Haar wavelet is the simplest and the fastest of all filter-bank-based wavelets to compute. For $N \times N$ 2-D signals, it can be computed digitally in $O(N^2)$ operations without multiplications and without roundoff error because the digital filter coefficients are integer powers of 2.

Designing practical optical wavelet transform systems provides a challenging goal for the optical engineering community. To achieve this goal, optical designers need to be aware of the performance of competing technologies.

References

1. H. H. Szu and H. J. Caulfield, "Guest editorial on wavelet transforms," *Opt. Eng.* **31**(9), 1823–1824 (1992).
2. H. J. Caulfield and H. H. Szu, "Parallel discrete and continuous wavelet transforms," *Opt. Eng.* **31**(9), 1835–1839 (1992).
3. I. Daubechies, "Orthonormal bases of compactly supported wavelets," *Comm. Pure Appl. Math.* **41**(7), 909–996 (1988).
4. O. Rioul and P. Duhamel, "Fast algorithms for discrete and continuous wavelet transforms," *IEEE Trans. Inf. Theory* **38**(2), 569–586 (1992).
5. M. Vetterli and C. Herley, "Wavelets and filter banks: theory and design," *IEEE Trans. Sig. Proc.* **40**(9), 2207–2232 (1992).
6. M. J. Shensa, "The discrete wavelet transform: wedding the à trous and Mallat algorithms," *IEEE Trans. Sig. Proc.* **40**(10), 2464–2482 (1992).
7. A. K. Soman and P. P. Vaidyanathan, "On orthonormal wavelets and paraunitary filter banks," *IEEE Trans. Sig. Proc.* **41**(3), 1170–1183 (1993).
8. O. Rioul and M. Vetterli, "Wavelets and signal processing," *IEEE Sig. Proc. Magazine,* pp. 14–38 (Oct. 1991).
9. E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable multiscale transforms," *IEEE Trans. Info. Theory* **38**(2), 587–607 (1992).
10. I. Daubechies, *Ten Lectures on Wavelets,* p. 100, Society for Industrial and Applied Mathematics, Philadelphia, PA (1992).
11. P. P. Vaidyanathan, "On coefficient-quantization and computational roundoff effects in lossless multirate filter banks," *IEEE Trans. Sig. Proc.* **39**(4), 1006–1008 (1991).
12. D. L. Flannery and J. L. Horner, "Fourier optical signal processors," *Proc. IEEE* **77**(10), 1511–1527 (1989).
13. S. Shen, S. Magar, R. Aguilar, G. Luikuo, M. Fleming, K. Rishavy, K. Murphy, and C. Furman, "A high performance CMOS chipset for FFT processors," *Proc. 1988 IEEE Intl. Conf. Computer Design: VLSI in Computers & Processors,* pp. 578–581 (1988).
14. J. S. Lim, *Two-Dimensional Signal and Image Processing,* pp. 163–166, Prentice Hall, Englewood Cliffs, NJ (1990).
15. M. Feibus and M. Slater, "Pentium power," *PC Magazine* **12**(8), 108–120 (1993).
16. R. Comerford, "How DEC developed Alpha," *IEEE Spectrum* **29**(7), 26–31 (1992).
17. R. K. Scannel and J. K. Hagge, "Development of a multichip module DSP," *IEEE Computer,* pp. 13–21 (April 1993).
18. B. Prince and R. H. W. Salters, "ICs going on a 3-V diet," *IEEE Spectrum,* pp. 22–25 (May 1992).
19. A. H. Tewfik, D. Sinha, and P. Jorgensen, "On the optimal choice of a wavelet for signal representation," *IEEE Trans. Info. Theory* **38**(2), 747–765 (1992).
20. J. Kovacevic and M. Vetterli, "Nonseparable multidimensional perfect reconstruction filter banks and wavelet bases for R^n," *IEEE Trans. Info. Theory* **38**(2), 533–555 (1992).
21. J. Kovacevic and M. Vetterli, "Perfect reconstruction filter banks with

arbitrary rational sampling rates," Technical Report No. CU/CTR/TR 233-91-14, Columbia University, Center for Telecommunications Research (1990).
22. T. J. Burns, K. H. Fielding, S. K. Rogers, S. D. Pinski, and D. W. Ruck, "Optical Haar wavelet transform," *Opt. Eng.* **31**(9), 1852–1858 (1992).
23. X. Yang, H. H. Szu, Y. Sheng, and H. J. Caulfield, "Optical Haar wavelet transform of binary images," *Opt. Eng.* **31**(9), 1846–1851 (1992).

Response

Invariant analog wavelets

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1 Introduction

We appreciate Dr. Duell's constructive comments in "Comment on 'Special section on wavelet transforms': Optical versus digital implementation of the wavelet transform." These comments are both general and specific. We would like to offer a reply to both.

We begin with the general "optical versus digital" debate. One of us is on record as substantially agreeing.[1] Just because something *can* be done optically does not mean that it *should* be done optically. Furthermore, the burden of proof is on optics. We further agree that the case is hardest to make for Fourier transformation. In our special section, we included papers that did not even attempt that burden of proof, because it is traditional in our field to do so. Whether that tradition should change is a topic worthy of serious discussion. We can defend both sides, so we are inclined not to fix what is not obviously broken.

Concerning several specific comments, we want to offer a reply. We made the following statement just for a comparison between shift-invariant optics and shift-sensitive digital processors: "A slight error in the [discrete] digital computing of shift variables can produce a large error in wavelet coefficients."[2]

We quote Dr. Duell's statement: "It is well known that the discrete wavelet coefficients are different for a signal and a shifted version of that signal. . . . This property in no way affects the *accuracy* of the transform. The transform coefficients of the original signal reconstruct the original signal, and the transform coefficients of the shifted signal reconstruct the shifted signal."

2 Efficiency in Classifications Versus Representations

We disagree with Dr. Duell's statement in defending the agreed shift-sensitive discrete wavelet transform (WT) for its accuracy in reconstruction. It is always the efficiency at ques-

tion. If one is willing to take as many terms as necessary, any complete set of basis is exactly equivalent to one another. The precise reason that WT is more powerful than Fourier transform (FT) in most applications is that for almost all interesting cases signals are localized but noise is global. WT offers a matching local basis that consequently requires a small number of expansion coefficients, each suffering less noise contamination. We call attention to the fact that the majority of applications in optics concern pattern classification/recognition (e.g., VanderLugt, inverse filters, Wiener, synthetic discriminant, scale invariant, etc., as opposed to data reconstruction or representation) of which invariant wavelet feature extraction seems to be one of the central strategies[3–5] performed by analog methods. Dr. Duell seems to confuse or ignore the important difference[6] between classification (which optics performs based on analog WT coefficients) and representation (which digital data compression often performs). Although the inverse WT can always reproduce the original result despite the shift-sensitive digital processing, it is not acceptable for any multiple resolution (MR) pattern recognition (PR). A signal based on its finite MR set of coefficients has the potential to be wrongly classified as a different class from that of the shifted version. In other words, we maintain that both digital and analog PR demand shift-invariant features. Our point is that this shift invariance is done readily by Fourier optics, with a possible additional bonus that the Fourier deconvolution theorem is often used to perform the WT inner product operation in the Fourier domain [for the saving of order $(N \log N)$ for the usual $O(N^2)$ linear matrix vector transfer].

3 Mathematics Behind Invariant Analog Wavelets

The FT is known to be an angle-preserving, or conformal, mapping. This suggests the design of optical wedge-ring detectors in the Fourier plane. This device works for scale-rotation invariance based on shift invariance because, despite the motion of the object, the square-law detectors guarantee the object centroid alignment at the origin of the Fourier domain. This shift invariance is mathematically based on the gauge freedom, namely, the modulus-invariant phase information in the following straightforward inner product mathematics denoted by brackets:

$$|FT_f\{g(t)\}|^2 \equiv |[e_f(t),g(t)]|^2 \equiv |G(f)|^2$$

$$= |G(f) \exp(-2\pi jfb)|^2$$

$$= \left| \int_{-\infty}^{+\infty} \exp(-2\pi jft)g(t-b) \, dt \right|^2 , \quad (1)$$

where a special WT basis turns out to be the sinusoidal basis $e_f(t) = \exp(-2\pi jft)$ known historically as FT.

A continuous version is obviously shift invariant in an unbounded domain:

$$WT_{ab}\{s(t)\} \equiv [h_{ab}(t),s(t)] = \int_{-\infty}^{+\infty} h_{ab}^*(t)s(t) \, dt \equiv W(a,b) . \quad (2)$$

As in Eq. (1), the inverse of Fourier frequency $1/f$ is related to the scale parameter a, and b is the shift parameter in question. This unified viewpoint that FT is a special case of WT

was actually exploited to combine the matched filter with the wavelet feature extraction filter into one filter operation for light efficiency in optical pattern recognition.[5] The additional mathematics in WT give us the freedom to treat the scale parameter a, the time t, and the shift parameter b in a self-consistent affine manner: $t' = (t-b)/a$. This helps us to generate basis functions $h_{ab}(t) = h(t')$ from an admissible mother function $h(t)$ that satisfies the zero area and finite energy condition, rather than by means of the Fourier harmonics $n2\pi f$ that disregard Gibb's overshooting phenomena and the locality of a signal in time t.

Recently, Szu et al. have shown[3–5] that the invariant property of the analog WT is based on the linear superposition principle of the intrinsic scaling law of the time-scale joint representation (TSJR) domain (a,b) (rather than the second-order Wigner distribution convolution and Woodward correlation time-frequency joint representation). The idea is simple. To investigate the invariant WT is to compute the WT of various scales of the identical signal. Hopefully, those scale-related WT coefficients organize themselves in such a fashion that they can be easily collected to produce scale-invariant features. This turns out to be a wedge shape, as follows. Let a generic signal under additive white noise be given by

$$s_i'(t') = s_i(\alpha_i t) + n(t) \; ; \quad i = 1, 2, \dots , \quad (3)$$

where the unknown scales α's (suppressing class index i) are equivalent to the unknown frequency compaction or hopping of similar waveforms $s(t)$'s. The associated WT coefficient denoted by the prime is computed as

$$W'(a,b) \equiv \int_{-\infty}^{+\infty} dt s'(t)h^*[(t-b)/a]/a$$

$$= \int_{-\infty}^{+\infty} dt' s'(t')h^*[(t'-b')/a']/a' . \quad (4)$$

Use is made of the change of variables: $t' = \alpha t$, $a' = \alpha a$, and $b' = \alpha b$, and Eq. (4) becomes exactly equal to the original $W(a,b)$ located radially by a factor of α in both the a and b planes [compare Eq. (31) in Ref. 4 for the conventional wavelet normalization of inverse square-root a], plus noise:

$$W'(a,b) = W(\alpha a, \alpha b) + \text{noise} . \quad (5)$$

Consequently, an optical wedge filter in the wavelet (a,b) domain designed by Szu et al. was used to capture the shift-scale information to produce an invariant PR through a simple neural network.

More generally, it is difficult to imagine any digital system performing a continuous transformation—Shannon, Whittaker, and Kotelnikov notwithstanding. We argue that this is a natural place for analog optics.

4 Conclusion

Finally, a great bulk of the papers in optics perform wavelet decomposition on 2-D images producing 4-D outputs. The data throughput is I/O limited in most of these cases. This puts us right back with the Fourier optics case in which electronics wins for small input scenes, optics wins for large-enough scenes, and the dividing line is changeable and dis-

putable. Our understanding of 2-D "fast wavelet transforms" is that they are nowhere near commercial availability as chips. If this is so, optics has a significant advantage for now. How large or small that advantage is and how the two approaches will evolve in the future are legitimate subjects of discussion. Dr. Duell has offered his opinions and backup arguments, which we appreciate and take seriously. We have offered ours, which are not entirely in disagreement. This is an important issue that is worthy of more discussion from all sides. In the near future, more adaptive iterative schemes that are implementable in real-time optoelectronic processors are anticipated for choosing data-driven appropriate mother wavelets by means of adaptive neural networks or variational superposition techniques to help solve invariant PR problems. In fact, this adaptivity will be the theme of another special section on adaptive wavelet transforms in the July 1994 issue of *Optical Engineering*.

References

1. H. J. Caulfield, "Pacifist's guide to optical computers," in *Optical Info. Proc. Syst. and Architectures III*, B. Javidi, Ed., *Proc. SPIE* **1564**, 632 (1991).
2. H. Szu and H. J. Caulfield, "Guest editorial on wavelet transforms," *Opt. Eng.* **31**(9), 1823–1824 (1992).
3. H. Szu, X. Yang, B. Telfer, and Y. Sheng, "Neural network and wavelet transform for scale-invariant data classification," *Phys. Rev. E* **48**(2), 1497–1501 (1993).
4. H. Szu, Y. Sheng, and J. Chen, "Wavelet transform as a bank of matched filters," *Appl. Opt.* **31**(17), 3267–3277 (June 1992).
5. Y. Sheng, D. Roberge, H. Szu, and T. Lu, "Optical wavelet matched filters for shift-invariant pattern recognition," *Opt. Lett.* **18**(4), 299–301 (Feb. 1993).
6. B. Telfer and H. Szu, "Implementing the minimun-misclassification-error energy function for target recognition," in *Intl. Joint Conf. Neural Network 92*, Vol. IV, Baltimore, MD, pp. IV-214–IV-219 (1992).