Research on optimal water level algorithm based on the Great Lakes

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ABSTRACT

In this paper, we establish multi-objective optimization model and water level prediction model through multi-task learning algorithm, entropy weight analytic hierarchy process (AHP), convolutional neural network (CNN) and Long Short-Term Memory (LSTM) neural network, etc., and calculated weights of 0.665, 0.761, 0.866, and 0.122 for water level elevation, river flow, cyclic variation, and water level stabilization, respectively, and Lake Ontario's optimal water level of 74.926 m.

Keywords: CNN, LSTM, optimum water level, multi-objective optimization, AHP

1. INTRODUCTION

The Great Lakes are the largest group of freshwater lakes in the United States and Canada and have a significant impact on the major cities, climate and local weather conditions in both countries[1]. The lake water level is affected by many factors, including temperature, wind, tide, precipitation, evaporation, sounding, river flow, reservoir policy, seasonal cycle and long-term climate change, etc. The current management structure is unable to cope with the highly uncertain impact of climate change[2]. We set up a multi-objective optimization model based on the combination of economic and ecological benefits and genetic algorithm to obtain the optimal water level of the Great Lakes at any time of the year.

2. DATA PREPROCESSING

It can be seen from the attached excel data table that the inflow, outflow and water level[3] of 11 lakes are respectively given. The data were combined to clearly observe the differences in discharge between lakes and facilitate the use of the following data. The monthly discharge of lakes in each year was obtained as shown in Table 1.

Table 1. Summary table of lakes.

	Superior	**Ontario**	**…**	**Erie**	**St. Clair**
00/1/31	183.2	74.5	…	173.8	174.6
00/2/29	183.1	74.45	…	173.8	174.4
…	…	…		…	…
22/10/31	183.6	74.41	…	174.3	175.3
22/11/30	183.6	74.4	…	174.2	175.1
22/12/31	183.57	74.47	…	174.2	175.04

We defined six categories of stakeholders: coastal residents, environmentalists, marina managers, shipping companies, hydroelectric companies, and recreational participants, and analyzed the Great Lakes system[4] in terms of optimal water level range, benefits, and costs to combine these six categories of stakeholders to obtain Figure 1.

We therefore quantified the needs of the six categories of stakeholders delineated for the Great Lakes in terms of optimal water levels, benefits, and costs for further study.

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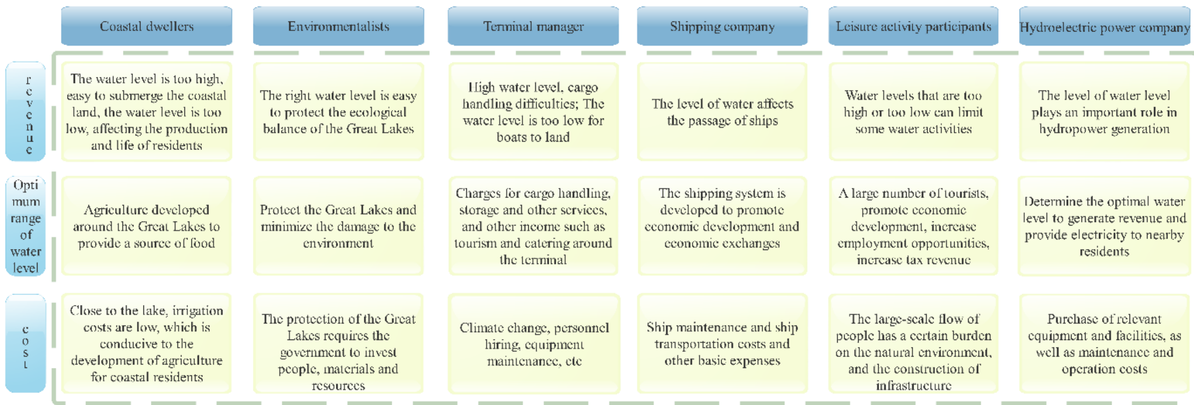

Figure 1. Stakeholder requirements in the Great Lakes system.

3. MODEL ESTABLISHMENT AND SOLUTION

According to the requirement quantification of stakeholders, three important indicators of the best water level, income and cost are obtained, and there is certain correlation and conflict among these indicators. We use Criteria Importance Though Intercrieria Correlation to empower metrics and prioritize stakeholders based on the end result. There are m targets to be evaluated and *n* evaluation indicators. The original index data matrix is composed as follows:

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix} \tag{1}$$

Among them, represents the value of the *j* evaluation index in the *i* evaluation objective.

Let the evaluation decision matrix of multi-attribute evaluation problem[5] be $W = \{w_1, w_2, w_3, \cdots, w_s\}$, The problem involves the indicator set being. The *i* evaluation objective is *w*; The index value of the *J* evaluation index P_j is denoted as b_{ij} $(i = 1, 2, \cdots, s\ ; j = 1, 2, \cdots, n)$.

In summary, the priority order is shipping companies>leisure activity participants>residents>environmentalists>hydro power companies>terminal managers.

We have established two objective functions, economic efficiency and ecological efficiency. The economic benefit maximizes the interests of all parties, and the ecological benefit ensures the optimal water level. We have established two objective functions, economic efficiency and ecological efficiency. The economic benefit maximizes the interests of all parties, and the ecological benefit ensures the optimal water level. Establishment of the objective function:

T represents the total month of the year, *T*=12; *t* represents the months of the year. Q_i^t represents the income of the first group of stakeholders for month *t*, H_i^t represents the cost of the first stakeholder in month t($i = 1, 2, 3, \cdots, 6$).

$$\bar{z} = \frac{5}{8}\left[z_0 \Delta t_1 + z_1(\Delta t_1 + \Delta t_2) + z_2(\Delta t_2 + \Delta t_3) + \cdots + z_{n-1}(\Delta t_{n-1} + \Delta t_n) + z_0 \Delta t_n\right] \tag{2}$$

$z_0, z_1, \cdots, z_n$ represents the observed water level of each measurement, Unit m; $\Delta t_1, \Delta t_2, \cdots, \Delta t_n$ represents the time interval between adjacent measurements, Unit h; $\bar{z}$ represents the monthly average optimal water level.

High water levels in lakes move to lower levels, and even normal variations in water levels of 2 feet to 3 feet can have an impact on stakeholders. In order to ensure optimal water levels in each area, the flow should be limited, and the flow is solved by geometrically approximating the body of the lake as a cone:

$$\begin{cases} \frac{d}{dt}v_1 = 0 - f & \frac{d}{dt}v_2 = f_1 - f_{21} \\ \frac{d}{dt}v_3 = f_2 - f_3 & \frac{d}{dt}v_4 = f_3 - f \end{cases} \tag{3}$$

Water consumption by coastal users:

$$\left(\sum_{j=1}^{n} \left| \sum a_j x + b_j \right|^2 \right)^{\frac{1}{2}} < c_j x + d_j \ (j = 1,2,3,...,m) \tag{4}$$

The dam controls the maximum water level

$$max\ W > w_p \ \ (w_p \text{ is the highest water level in the lake area}) \tag{5}$$

The maximum draft of the ship[6]:

$$max\ F < f_{max} \ (f_{max} \text{ is the maximum flow rate}) \tag{6}$$

According to the model established above, multi-objective genetic algorithm is used to solve it.

On the basis of meeting the interests of all parties, the optimal water level results are shown in Table 2:

Table 2. Optimal lake water level results.

	Michigan/Huron	St. Clair	Ontario	Superior	Erie
00/1/31	175.87	174.75	74.5	183.16	174.87
00/2/29	175.87	174.44	74.45	183.08	173.75
00/3/31	175.9	174.66	74.58	183.08	173.93
00/4/30	175.92	174.76	74.82	183.12	173.95
…	…	…	…	…	…
22/10/31	176.6	175.25	74.41	183.56	174.32
22/11/30	176.55	175.14	74.4	183.6	174.25
22/12/31	176.46	175.12	74.47	184.67	174.2

It can be seen from the table that the optimal water level of the lake remains stable within a certain time range. During this time period, the needs of various stakeholders are balanced to obtain the optimal water level of the lake and promote the development of other aspects of the Great Lake's waters such as fishing, irrigation and shipping.

As can be seen, the water level of the lake is affected by natural recharge[7], which is reflected in precipitation, snowmelt and evaporation, and by the regulation of DAMS, which control the outflow of Lakes Ontario and Superior. Using multi-task learning algorithms, it can dynamically adjust and update the demand in real time, thus maintaining the water level in each month's time period.

Taking Lake Superior as an example, the correlation between inflow and outflow values in the lake was extracted.

Let M stand for prediction task, $X_{I,M}^e$ represents the outflow or inflow from Lake I during the 1 month interval, X_M^e represents the collection of lake discharge in 1 month. Therefore, new flow data for each month is projected, taking into account the lake flow $\left[X_M^{e-q}, \cdots, X_M^e \right]$ at monthly intervals throughout the year and external influences λ.

$$\left[X_M^{e+1}, \cdots, X_M^{e+E} \right] = F\left(\left[X_M^{e-q}, \cdots, X_M^e \right], \lambda \right) \tag{7}$$

The five lakes[8] flow into the Atlantic Ocean from west to east, and the terrain is high in the west and low in the east. The delayed effect of flowing water caused by the location and slope of DAMS will also affect the level of water level. The convolutional neural network can be used to obtain the spatial relationship of inflow and outflow of each lake, and the residual network calculation formula is as follows:

$$X_{r+1} = R\left(X_r, Q_r\right) + X_r \tag{8}$$

X_{r+1}, X_r respectively represents the input and output of the residual unit.

Q_r represents a learnable parameter, R represents the residual function.

It can be seen from the appendix that snowmelt water supplies the lake in spring; The water level reaches its highest in summer. The water level drops again in autumn. For this purpose, LSTM neural network is used.

Combining the above information and focusing on the inflow and outflow data of the Great Lakes, a multi-task learning algorithm is established to maintain the optimal water level.

The final projected monthly optimum levels for the Great Lakes are shown in Table 3:

Table 3. Best monthly water table for the Great Lakes.

Time \ Lake	**Michigan/Huron**	**St. Clair**	**Ontario**	**Superior**	**Erie**
01/1/31	175.97	174.71	74.49	182.98	173.75
01/2/28	175.78	174.69	74.55	182.94	173.84
01/3/31	175.78	174.7	74.65	182.91	173.94
01/4/30	175.85	174.74	74.84	183.01	174.02
…	…	…	…	…	…
21/10/31	176.9	175.61	74.78	183.48	174.67
21/11/30	176.77	175.48	74.86	183.38	174.63
21/12/31	176.69	175.42	74.87	183.31	174.58

It can be seen from the table that the results obtained by the second question are ideal, with high prediction accuracy, little difference from the reality, and there is a stable period.

We compare the difference between the water level obtained by the control algorithm and the optimal water level, draw thermal maps of the two kinds of data for visualization, and calculate the weight of each indicator by combining the entropy weight analytic hierarchy process[9], thus demonstrating the superiority of the control algorithm and making all stake-holders more satisfied.

Heat maps are drawn respectively according to the water level data obtained in Questions 1 and 2 in 2017, as shown in Figure 2.

1	0.97	0.29	0.6	0.89	0.75	0.71	0.77	0.62	0.88	0.66
0.97	1	0.18	0.68	0.79	0.67	0.77	0.68	0.92	-0.17	0.53
0.29	0.18	1	-0.06	0.6	0.53	-0.07	-0.17	0.01	-0.18	0.35
0.6	0.68	-0.06	1	0.37	0.2	0.9	0.83	0.84	0.48	0.01
0.89	0.79	0.6	0.37	1	0.87	0.43	0.32	0.64	-0.36	0.64
0.75	0.67	0.53	0.2	0.87	1	0.22	0.05	0.43	-0.61	0.58
0.71	0.77	-0.07	0.9	0.43	0.22	1	0.97	0.93	0.42	0.17
0.62	0.68	-0.17	0.83	0.32	0.05	0.97	1	0.87	0.5	0.15
0.88	0.92	0.01	0.84	0.64	0.43	0.93	0.87	1	0.13	0.4
-0.24	-0.17	-0.18	0.48	-0.36	-0.61	0.42	0.5	0.13	1	-0.54
0.66	0.53	0.35	0.01	0.64	0.58	0.17	0.15	0.4	-0.54	1

(a)

1	0.95	0.3	0.62	0.87	0.78	0.73	0.77	0.67	0.87	0.64
0.95	1	0.19	0.58	0.79	0.67	0.87	0.68	0.94	-0.17	0.53
0.3	0.19	1	-0.06	0.6	0.53	-0.07	-0.2	0.01	-0.18	0.35
0.62	0.58	-0.06	1	0.37	0.2	0.9	0.84	0.84	0.48	0.01
0.87	0.79	0.6	0.37	1	0.87	0.43	0.32	0.64	-0.36	0.66
0.78	0.67	0.53	0.2	0.87	1	0.22	0.05	0.43	-0.61	0.56
0.73	0.87	-0.07	0.9	0.43	0.22	1	0.97	0.93	0.42	0.17
0.67	0.68	-0.2	0.84	0.32	0.05	0.97	1	0.87	0.5	0.16
0.87	0.94	0.01	0.84	0.64	0.43	0.93	0.87	1	0.13	0.43
-0.24	-0.17	-0.18	0.48	-0.36	-0.61	0.42	0.5	0.13	1	-0.45
0.64	0.53	0.35	0.01	0.66	0.56	0.17	0.16	0.43	-0.45	1

(b)

Figure 2. (a): Heat map of water level data in question 1; (b): Heat map of water level data in question 2.

It can be seen from the figure that the water level of each lake in question 1 varies within a certain range during the monthly time period in 2017.

It can be seen from the figure that the water level data of the lake obtained in question 2, combined with the thermal map, it can be seen that the water level data obtained in question 1 and 2 are not very different.

The use of entropy weight analytic hierarchy process (AHP) to calculate the weight needs to determine the indicators, and because the use of AHP needs to determine the target layer, scheme layer and criterion layer.

Step 1. Data standardization

Let the number of objects to be evaluated be *y* and the number of indicators be *k*, de-noted as $\alpha_1,\alpha_2,\cdots,\alpha_k$, The standardized data is denoted as $\beta_1,\beta_2,\cdots,\beta_k$, Then the standardized data formula is:

$$\beta_{ij}=\frac{\alpha_{ij}-\min(\alpha_{ij})}{\max(\alpha_j)-\min(\alpha_j)}\ (i=1,2,...,y\ ;j=1,2,...,k) \tag{9}$$

Step 2. The information entropy of the four indexes calculation

$$s_j=-\frac{1}{\ln(y)}\sum_{i=1}^{y}N_{ij}\ln(N_{ij})\ (i=1,2,...,y\ ;j=1,2,...,k) \tag{10}$$

Step 3. The weights of the four indicators calculation

$$w_j=\frac{1-s_j}{k}\ (j=1,2,...,k) \tag{11}$$

Finally, the weights of the four indicators are 0.665, 0.761, 0.866, and 0.122, respectively. It can be clearly seen that the superiority of the applied control algorithm in solving the water level is much higher than the value obtained without the control algorithm, reflecting the control algorithm applied in this paper, which can effectively adjust the water level according to the actual rainfall and other conditions to keep it within the best range. Question 4 requires an analysis of the sensitivity of environmental conditions. According to the information given by question stem, the environmental factors that cause disturbance to water level mainly include rainfall, snow cover, evaporation, erosion and ice jam. Assuming that these factors are independent of each other, Sobol global sensitivity analysis is used to do the sensitivity analysis[10] of input variables to explore the impact of environmental conditions on water level.

Sobol global sensitivity calculation function is as follows:

$$f(y)=f_0+\sum_{i=1}f_i(y_i)+\sum_{1\le i\le j\le n}f_{ij}(y_i,y_j)+\cdots+f_{1,2,\cdots,n}\left(y_1,...,y_n\right) \tag{12}$$

Among them, are different variables; f_0 is a constant term; *n* represents total number of variables.

Total variance of the input variable:

$$G=\int f^2(y)dy-f_0^2 \tag{13}$$

Sensitivity index:

$$S_{i1,i2,\cdots,is}=\frac{D_{i1,i2,...,is}}{D} \tag{14}$$

Sensitivity coefficient of input variable:

$$y_i\ S'=\sum_{\{i1,i2,\cdots,is\}}S_{i1,i2,\cdots,is} \tag{15}$$

The sensitivity analysis results obtained under each weather variable are shown in Figure 3:

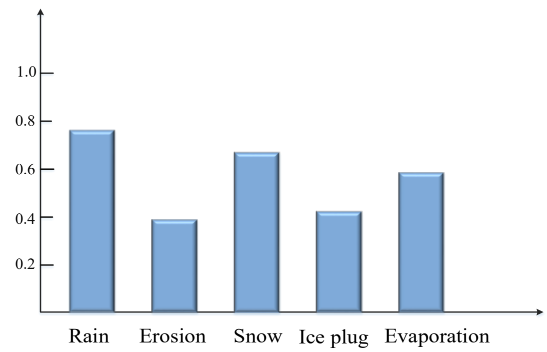

Figure 3. Sensitivity index result.

As can be seen from the figure, the sensitivity indexes of rainfall, snow cover, evaporation, erosion and ice jam on water level are all high, and the sensitivity of rainfall is the highest. These five aspects all play an important role in the change of lake water level and discharge[11].

For the sensitivity analysis of the above weather and environmental factors to water level, in order to make the results clearer and more understandable, the line chart of some lake water level changes is drawn as Figure 4:

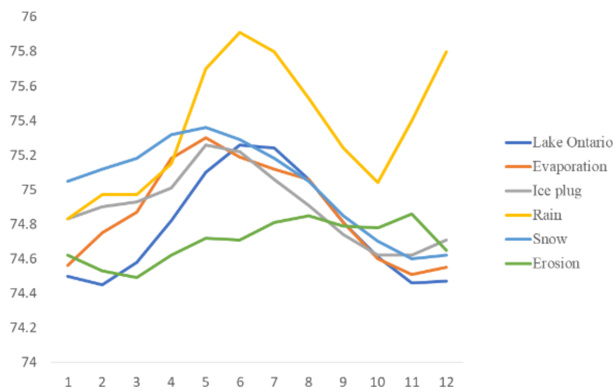

Figure 4. Line chart of water level under weather influence.

As can be seen from the figure, these five weather factors have an impact on the flow of lakes, resulting in some lakes with abnormally high-water levels or lower water levels. And rainfall is more important to the change of water level, and the sensitivity of evaporation is far behind rainfall. In order to analyze the relevant stakeholders and factors affecting Lake Ontario, we follow the model in Problem 1 and solve for the optimal level of Lake Ontario based on the subproblems controlling the level of Lake Ontario and the potential factors to improve the constraints .

In order to make the data more intuitive, the water level of Lake Ontario given in the appendix is compared with the optimal water level requested above, and the distribution map is drawn as Figure 5:

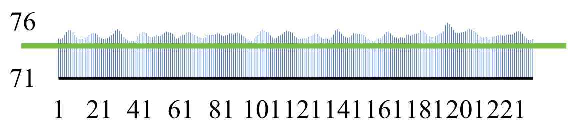

Figure 5. Water level distribution comparison map.

As can be seen from the figure, increasing the control and management of water level can effectively improve the demand degree of stakeholders. It is necessary to focus on the water level of Lake Ontario, and pay close attention to its water level changes, so that stakeholders can avoid losses in economy, security, traffic and entertainment. Actually, improve the relationship with stakeholders.

4. CONCLUSION

Strengths

(1) In problem 1, a multi-objective optimization model is established, which realizes the collaborative optimization of the objectives of multiple stakeholders and improves the efficiency to a certain extent.

(2) In problem 2, convolutional neural network algorithm is used to extract spatial features, effectively reducing the complexity of feature extraction and saving time;

(3) In problem 2, the multi-task learning algorithm is used to control the optimal water level, which has a high utilization of flow data and strong learning performance;

(4) In problem 3, the entropy weight analytic hierarchy process is used to calculate the weight of indicators, and the indicators affecting the optimal water level are qualitatively analyzed, which is comprehensive.

Weaknesses

(1) In problem 1, when the CRITIC weight method is used to obtain stakeholder demand indicators, if there is a strong correlation between indicators, the output results will be unstable;

(2) In problem4, when Sobol global sensitivity analysis is used to explore the sensitivity of water level to environmental conditions, there is a certain subjectivity.

Model improvement

For the first shortcoming, closely combined with the explanatory basis given in the appendix and water level data, judge the impact of environmental conditions, reduce subjectivity;

In view of the second shortcoming, pay attention to distinguish the correlation degree of the selected indicators, reduce the mutual influence, so as to improve the stability of the results.

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