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Range Dependence of Pulse Position Modulation in the Presence of Background Noise

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ABSTRACT

We identify the maximum attainable transmission rate of a noisy optical link employing the pulse position modulation (PPM) format with direct detection. We show that for a fixed background noise level it is possible to achieve the information rate directly proportional to the average detected signal power in the photon-starved regime. This implies inverse-square scaling with the distance, presenting a qualitative improvement over previously obtained estimates that scaled as the inverse of the fourth power of the distance. The necessary ingredients to achieve the improved mode of operation are the unrestricted optimization of the PPM order and the complete decoding of detection events to extract information from sequences containing multiple counts within one PPM frame. Importantly, information efficiency equivalent to high-order PPM formats can be attained using signals with uniformly distributed optical power processed with recently proposed structured optical receivers.
Keywords: photon-starved communication; pulse position modulation; range dependence; background noise

1. INTRODUCTION

Optical domain offers numerous benefits for deep-space communication.[1] The main advantage, when compared to radio frequencies, is a much larger available bandwidth. Furthermore, the use of laser sources allows for improved targeting of the signal power, greatly reducing diffraction losses resulting from the propagation through space. Other technical reasons, such as lesser size and weight of communication systems and relaxed regulatory issues also make optical communication a promising technology for space missions.

The standard approach to deep-space optical communication relies on the pulse position modulation (PPM) format which encodes information in the position of a light pulse in a sequence of otherwise empty time bins.[2] Without background noise, a light pulse impinging on a photodetector produces a photocount with a probability smaller than one, implying that some PPM symbols escape detection, as shown in Fig. 1(a). Such erasure events can be dealt with using standard error correcting codes.[3] It can be shown that with diminishing detected signal power, the PPM format optimized over its order (i.e. the symbol length) attains the information rate which has the same scaling in the power parameter as the ultimate quantum mechanical limit on the capacity of an optical channel.[4,5] The situation becomes more nuanced in the presence of background noise. Previous analyses indicated that in this case the information rate scales asymptotically as the inverse of the fourth power r^{-4} with the distance r covered by the optical link.[6,7] This is quadratically worse compared to what can be in principle achieved by radio frequency links, exhibiting r^{-2} inverse square scaling.[8] On the other hand, PPM can be considered as a restricted version of generalized on-off keying (OOK), where a pulse is sent in any time bin with a predefined probability satisfying the average power constraint. In the latter case it has been shown that in the noisy scenario the information rate can actually attain inverse-square scaling with the distance.[9] This leads to the question if analogous scaling can be also achieved with the PPM format. In this contribution we summarize the recently given[10] positive answer to this question, based on two assumptions. The first one is that the PPM order can be arbitrarily high. The second one is that information is extracted from all sequences of counts recorded within one PPM frame, i.e. that complete decoding is implemented. This is in contrast with a simple decoding approach, when frames containing only single counts are retained for processing and all other count combinations are treated as erasure events. There is a qualitative difference between information rates attainable with complete and simple decoding, which emphasizes the need for advanced error correction.

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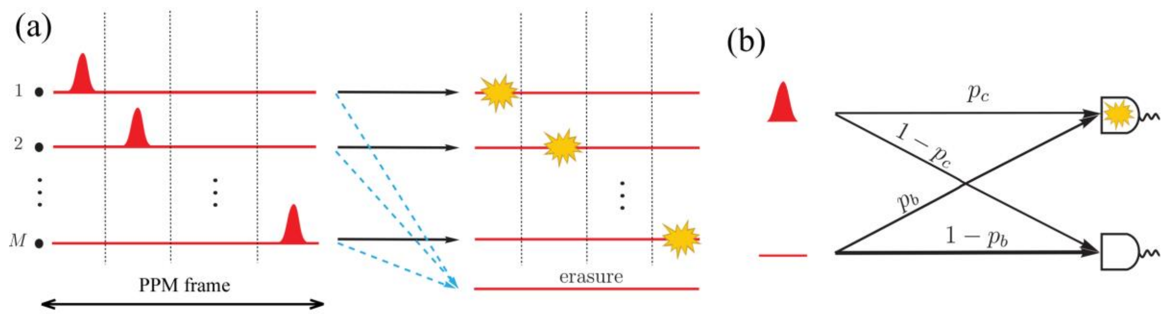

Figure 1. (a) Idealized PPM link without background noise, with M PPM symbols defined by the position of one light pulse within a frame of M time bins. The timing of the detector count allows for unambiguous identification of the input symbol, while no-count events are treated as erasures. (b) In the presence of background noise both an empty bin and a light pulse can produce a count with respective probabilities p_b and p_c.

2. SYSTEM DESCRIPTION

The essential parameters characterizing an optical link are the carrier frequency f_c, the emitted power P and the available bandwidth B. The last quantity defines the minimum time bin duration as $\Delta t = B^{-1}$. The receiver efficiency η_{rx} and the transmission of the optical channel can be combined into a single parameter

$$\eta = \eta_{\text{rx}} \left(\frac{\pi f_c D_{\text{tx}} D_{\text{rx}}}{4cr} \right)^2 \tag{1}$$

Here D_{tx} and D_{rx} stand for diameters of the transmitter and the receiver telescopes, r is the distance between the transmitter and the receiver and c is the speed of light in vacuum. The inverse-square scaling of η with r stems from diffraction losses. An important figure of merit is the average detected optical energy per time bin, expressed in the energy units hf_c of one photon at the carrier frequency, where h is Planck's constant:

$$n_a = \frac{\eta P \Delta t}{hf_c} = \frac{\eta P}{hf_c B} \tag{2}$$

This quantity enters the calculation of the Shannon mutual information I per time bin, which defines the maximum attainable transmission rate for a given modulation format and detection scheme. In the photon-starved regime, when $n_a \ll 1$, it is more convenient to analyze the photon information efficiency (PIE) given by PIE $= I/n_a$. Note that if in the asymptotic regime $n_a \to 0$ the PIE approaches a constant, the Shannon information is directly proportional to the average detected signal power n_a and hence exhibits inverse-square scaling with the distance r following Eqs. (1) and (2).

In the M-ary PPM format, the optical energy of an entire frame of M time bins is concentrated in a single light pulse. Thus it contains on average Mn_a photons. In the scenario without background noise, shown in Fig. 1(a), the probability of generating a count is $1 - \exp(-Mn_a)$ according to the standard theory of photodetection.[11] This figure defines the probability that a PPM symbol produces a count in the respective time bin. Otherwise no count is observed and the PPM symbol is erased with the probability $\exp(-Mn_a)$. We consider here a Geiger mode detector which provides only a binary response whether at least one photon has been registered or none at all. For simplicity, we neglect the detector dead time.

In the presence of background noise, a count can occur in any time bin according to the probability diagram in Fig. 1(b). For a background noise power n_b, measured in the same photon number units per time bin as the signal power, the probability of generating a count by an impinging pulse is $p_c = 1 - \exp(-Mn_a - n_b)$, while a count occurs in an empty bin with a probability $p_b = 1 - \exp(-n_b)$. These formulas are based on an assumption that background counts are generated by a Poissonian process independent from the optical signal. Importantly, counts may now occur in multiple bins within one PPM frame shown in Fig. 1(a). The most straightforward strategy would be to ignore frames containing multiple counts, treating them on equal footing with erasures. In this *simple decoding* approach the receiver returns one of $M + 1$ responses corresponding to individual PPM symbols plus an erasure, but the background noise results in a non-zero crossover probability between different symbols. The most sophisticated strategy would be to process all 2^M possible patterns that can be produced by the detector within one PPM frame, corresponding to *complete decoding*.

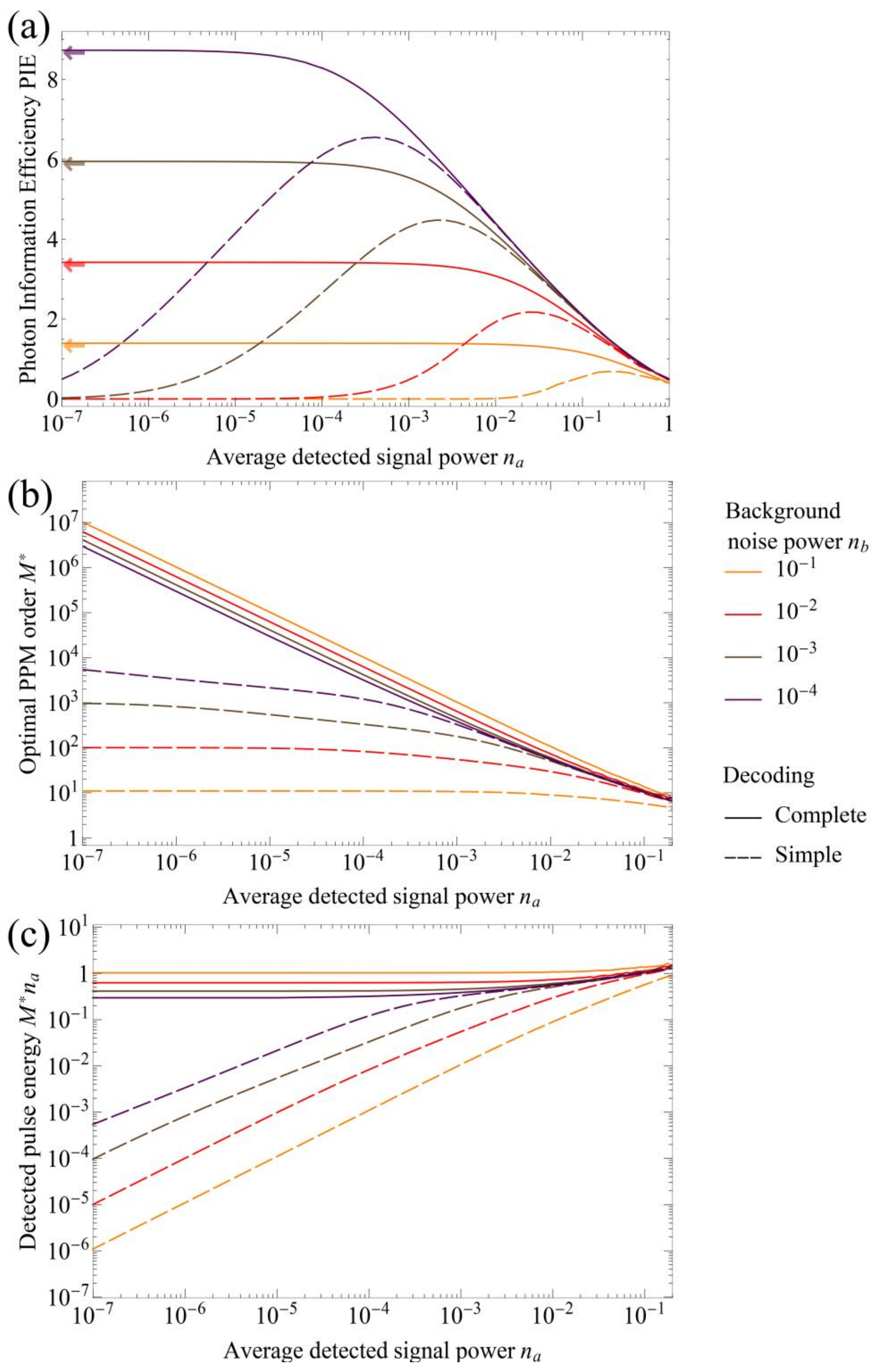

Figure 2. (a) Photon information efficiency (PIE) optimized over the PPM order as a function of the detected optical signal power n_a for the background noise power $n_b = 10^{-1}, 10^{-2}, 10^{-3}$ and 10^{-4} assuming complete decoding (solid lines) and simple decoding (dashed lines). The arrows on the left axis indicate the asymptotic values obtained from Eq. (3). (b) Optimal PPM order M^* and (c) the corresponding product $M^* n_a$ specifying the optimal detected pulse energy.

3. OPTIMIZATION

In Fig. 2(a) we show PIE optimized over the PPM order M taken to be a real parameter with a constraint $M \geq 2$ as a function of the detected optical power n_a for several values of the background noise power n_b. A stark difference between complete decoding and simple decoding scenarios is clearly seen. While in the case of complete decoding PIE attains a constant value in the limit $n_a \to 0$, it vanishes when simple decoding is used. It can be shown[10] that in the latter case PIE scales linearly with n_a. The enhancement offered by complete decoding requires unrestricted growth of the optimal PPM order M^* as depicted in Fig. 2(b). To a good approximation, M^* is inversely proportional to n_a. This is clearly seen in Fig. 2(c) showing the product $M^* n_a$, which specifies the mean photon number in the detected pulse for the optimal PPM order. As a rule of thumb, the pulse optical energy is in the range $10^{-1} - 10^{0}$. The simple intuition behind this result is that no matter how low the average detected signal power becomes, it should be concentrated in sufficiently few bins so that the pulses produce counts with high probability, well above the background count rate.

As shown in[10] using information theoretic results on the capacity per unit cost,[12] the asymptotic value of PIE for the complete decoding scenario can be found by solving an elementary single-parameter optimization problem defined as

$$\text{PIE} \to \max_{n_s} \left\{ \frac{1}{n_s} D(\exp(-n_s - n_b) || \exp(-n_b)) \right\} \tag{3}$$

In the above expression, $D(p||q)$ is the relative entropy between two binary random variables given explicitly by

$$D(p||q) = p \log_2 \frac{p}{q} + (1-p) \log_2 \frac{1-p}{1-q} \tag{4}$$

In Fig. 3(a) we plot the asymptotic PIE calculated according to Eq. (3) as a function of the background noise power n_b. The value of n_s maximizing the right hand side of Eq. (3) specifies the optimal pulse optical energy $M^* n_a$ in the asymptotic limit. This quantity is depicted in Fig. 3(b).

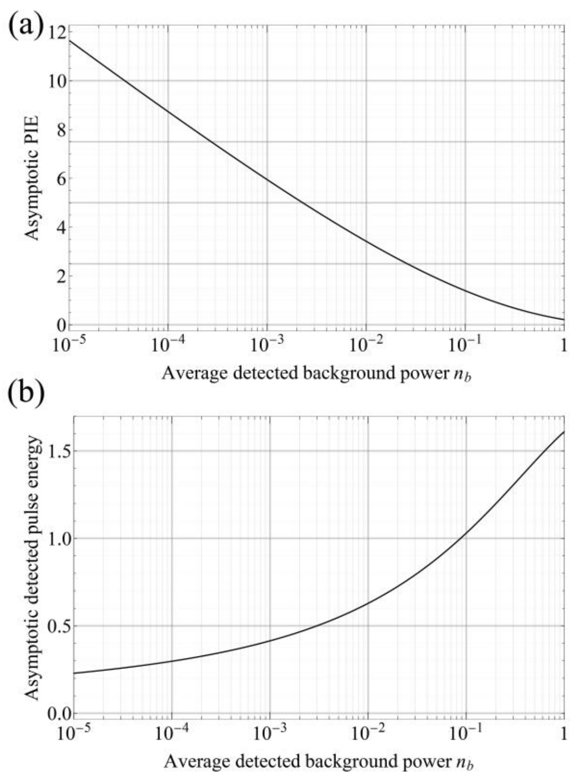

Figure 3. (a) The asymptotic value of the photon information efficiency in the limit $n_a \to 0$ calculated according to Eq. (3) as a function of the background noise power n_b. (b) The optimal detected optical energy $n_s = M^* n_a$ of the PPM pulse in the asymptotic limit $n_a \to 0$.

The information rate R for a link using a bandwidth B reads

$$R = B \cdot I = B \cdot \mathrm{PIE} \cdot n_a \tag{5}$$

It is seen that constant PIE makes the information rate directly proportional to the detected signal power n_a, which scales as r^{-2} according to Eqs. (1) and (2). Fig. 4(a) depicts the information rate for an exemplary deep space link as a function of the distance, evaluated for the carrier frequency $f_c = 2 \cdot 10^5\ GHz$, corresponding to the wavelength $\lambda = 1500\ nm$, the bandwidth $B = 2\ GHz$, the transmitter power $P = 4\ W$, the receiver efficiency $\eta_{rx} = 2.5\%$, and the transmitter and the receiver telescope diameters respectively $D_{tx} = 0.22\ m$ and $D_{rx} = 11.8\ m$. The numerical difference between complete and simple decoding is significant, for example at the distance $r = 10\ AU$ and background noise level $n_b = 10^{-1}$ the improvement is nearly hundredfold.

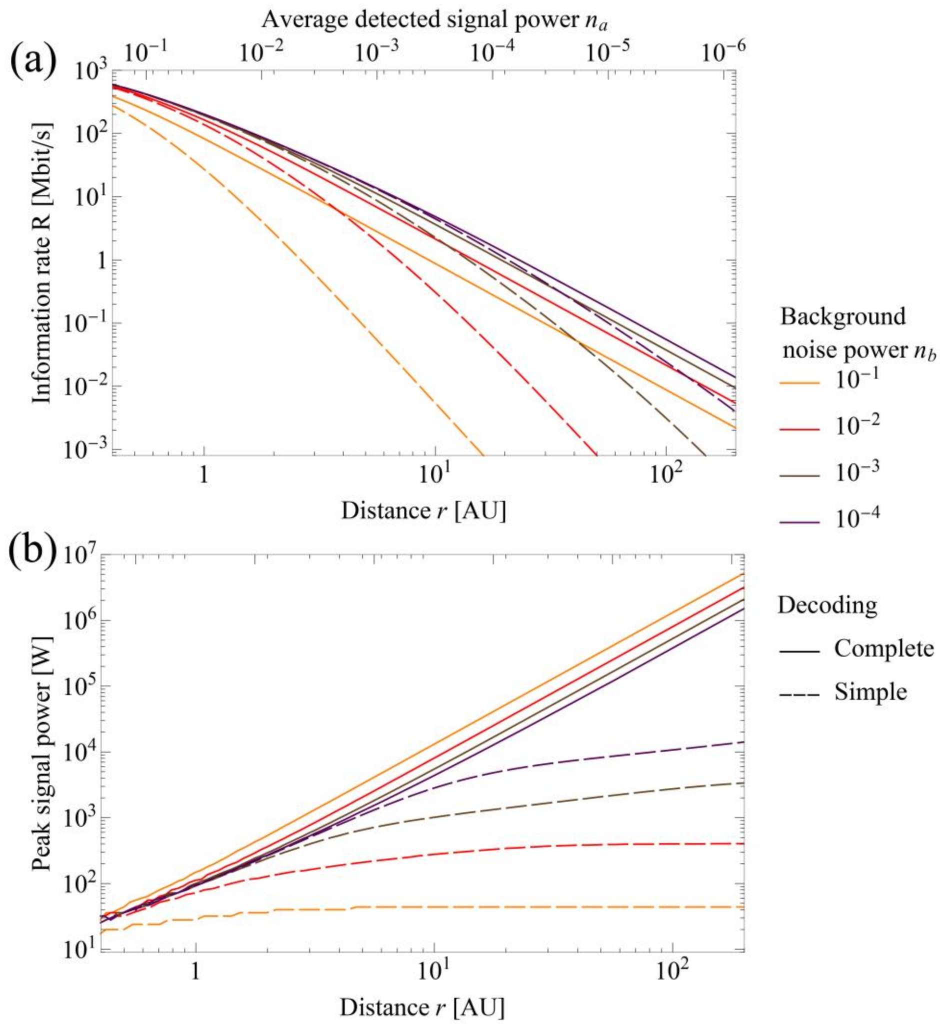

Figure 4. (a) The information rate of an optical PPM link optimized over the PPM order as a function of the link distance r for different levels of the background noise power n_b. The link parameters are: carrier frequency $f_c = 2 \cdot 10^5\ GHz$; bandwidth $B = 2\ GHz$; transmitter power $P = 4\ W$; receiver efficiency $\eta_{rx} = 2.5\%$; transmitter telescope diameter $D_{tx} = 0.22\ m$; receiver telescope diameter $D_{rx} = 11.8\ m$. The complete decoding scenario (solid lines) is compared to simple decoding (dashed lines). For clarity, the horizontal scale is specified both in the physical distance (bottom) and the average detected photon number per time bin (top). (b) The required peak power assuming a rectangular shape of the PPM pulse filling the entire time bin duration $\Delta t = B^{-1}$.

4. OUTLOOK

The hardware challenge to reach the r^{-2} scaling of the information rate is to generate signal with unboundedly growing peak-to-average signal ratio. Fig. 4(b) shows the required peak signal power assuming a simple rectangular pulse shape filling the entire time bin. A possible remedy to this problem is to produce a signal with uniformly distributed optical power, using e.g. using words composed from the binary phase shift keying (BPSK) format and to convert them into the high-order PPM format at the ground station using a structured optical receiver.[13] Importantly, scalable designs for structured optical receivers have been described,[14] in which the number of required optical elements scales logarithmically with the PPM order. The physical principle of these receivers is to combine coherently the optical energy spread over multiple time bins into a single pulse with a well-defined arrival time. Hardware implementation of a very high order PPM format or its equivalent needs to be matched by efficient software for error correction capable of processing events containing multiple counts within individual PPM frames.

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