

Field Guide to

Laser Cooling Methods

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Table of Contents

Preface	xi
Glossary	xiii
Introduction	1
Brief History of Laser Cooling	1
Laser Cooling and Trapping of Atoms and Ions	2
Methods of Laser Cooling	3
Potential and Kinetic Energy	4
Conservation Laws	5
Gases, Liquids, Solids, and Plasma	6
The Zeroth Law of Thermodynamics	7
Temperature and Thermometers	8
Maxwell–Boltzmann Distribution	9
Thermal Physics	10
Matter and Light	11
Angular Momentum of Atoms	11
Multi-electron Atoms	12
Density Matrix	13
Interaction of Light with a Two-Energy-Level System	14
Rabi Frequency	15
Dressed Atom	16
The Doppler Effect	17
The Stark Effect	18
The Zeeman Effect	19
Emission-Broadening Processes	20
Homogeneous (Collisional) Broadening	21
Inhomogeneous Broadening	22
Electric Susceptibility	23
Radiative Force	24
Doppler Cooling	25
Doppler Cooling	25
Semi-classical Treatment of Doppler Cooling: Force	26
Semi-classical Treatment of Doppler Cooling: Temperature	27

Table of Contents

Sisyphus Cooling	28
Motivation for Sisyphus Cooling	28
Sisyphus Cooling	29
Traps	30
Earnshaw's Theorem	30
Traps	31
High-Field and Low-Field Seekers	32
Optical Dipole Trap	33
Optical Dipole Trap	33
Heating Mechanism	34
Optical Lattices	35
Optical Lattice versus Solid State Crystal Lattice	36
Magnetic Trap	37
Magnetic Fields of Circular Current Loop	37
Landau–Zener Problem	38
Majorana's Spin-Flip Transitions	39
Magnetic Trap	40
Quadrupole Trap	41
Time-Orbiting-Potential Trap	42
Ioffe–Pritchard Trap	43
Comparison of Trap Types	44
Magneto-optical Trap	45
Magneto-optical Trap (MOT)	45
MOT Loading	46
Loading Cooling Methods	47
Cooling-Free Loading Methods	48
Collisions	49
Collisions	49
Scattering Amplitude	50
Cross Sections	51
Radial Potentials and Partial Wave Expansion	52
Scattering of Identical Particles	53
Pseudo-potential	54

Table of Contents

Evaporative Cooling	55
Trapping Charged Particles	56
Trapping Charged Particles	56
Paul Traps	57
Paul's Discovery	58
Potentials in Quadrupole 3D and 2D Paul Traps	59
The Mathieu Equation	60
Cyclotron Motion	61
Penning Traps	62
Penning Trap Frequencies	63
Other Ion Trap Designs	64
Quantum Jumps	65
Trapped Ions and Their Applications	66
Sub-recoil Cooling	67
Motivation for Sub-recoil Cooling	67
Spin-Polarized Atoms	68
Optical Pumping and Selection Rules	69
Dark States and Coherent Population Trapping	70
Random Walk	71
Velocity-Selective Coherent Population Trapping	72
Free-Space Raman Cooling	74
Interaction with a Trapped Atom	75
Lamb–Dicke Regime	76
Weak and Strong Confinement	77
Motional Sideband Excitation	78
Raman Sideband Cooling	79
Degenerate Raman Sideband Cooling	80
Electromagnetically Induced Transparency Cooling	81
Bose–Einstein Condensate	82
Thermal Distribution Functions	82
Matter Waves	83
Bosons and Fermions	84
Bose–Einstein Condensate	85
Alkali Atoms	86

Table of Contents

Diagnostics	87
Atom–Light Interactions	87
Fluorescence Imaging	88
Absorption Imaging	89
Fourier-Filtering Techniques	90
Dispersive Dark-Ground Imaging	91
Phase-Contrast Imaging	92
Time-of-Flight Method	93
Rare-Earth-Doped Solids	94
Rare-Earth Ions	94
Ions in Solids	95
Judd–Ofelt Theory	96
Radiative Transitions	97
Einstein B Coefficient	98
Cross Sections	99
Phonons	100
Vibrations in 1D Periodic Systems	100
Phonons	101
Nonradiative Transitions	102
Ion–Ion Interaction	103
Resonant Radiative and Nonradiative Transfer	103
Another Energy Transfer Process	104
Laser Cooling of Rare-Earth-Doped Solids	105
Pringsheim’s Cooling	105
Thermodynamics of Optical Cooling of Solids	106
Laser Cooling with ASF in Different Systems	107
Electrons in Stark Sublevels of Rare-Earth Ions	108
Two-Level Model of RE-Doped Solids	109
Laser Cooling in Two-Level RE-Doped Systems	110
Laser Cooling in Ideal RE-Doped Hosts	111
Laser Cooling in Real RE-Doped Hosts	112
Host Materials for Laser Cooling	113
Obstacles to Laser Cooling of RE-Doped Solids	114
Optimization of RE-Doped Samples for Laser Cooling	115

Table of Contents

Achievements in Laser Cooling of RE-Doped Samples	116
Optical Cavities for Laser Cooling	117
Thermal Links	118
Radiation-Balanced Laser	119
Raman Cooling of Solids	120
Spontaneous Raman Scattering	120
Stimulated Raman Scattering	121
Raman Cooling of Solids	122
Raman Laser with Heat Mitigation	123
Laser Cooling with STIRAP	124
Stimulated Raman Adiabatic Passage (STIRAP)	124
Laser Cooling with STIRAP	125
Brillouin Cooling	126
Hybrid Laser Cooling	127
Equation Summary	128
Bibliography	133
Index	136

Preface

Cooling or refrigeration is based on heat removal and dates back thousands of years to when people tried to preserve their food using ice and snow. The laser—a groundbreaking scientific achievement of the 20th century— has revolutionized the cooling process. The advent of lasers brought laser cooling, also known as optical refrigeration, into existence. Today, laser cooling and its applications represent one of the major subfields of atomic, molecular, and solid state physics.

This Field Guide provides an overview of the basic principles of laser cooling of atoms, ions, nanoparticles, and solids, including Doppler cooling, polarization gradient cooling, different sub-recoil schemes of laser cooling, forced evaporation, laser cooling with anti-Stokes fluorescence, hybrid laser cooling, and Raman and Brillouin cooling. It also covers radiation-balanced lasers and Raman lasers with heat mitigation, and considers the basic principles of optical dipole traps, magnetic traps, and magneto-optical traps. This Field Guide will serve both to introduce students, scientists, and engineers to this exciting field, and to provide a quick reference guide for the essential math and science.

I would like to thank SPIE Press Manager Timothy Lamkins and Series Editor John Greivenkamp for the opportunity to write a Field Guide for one of the most interesting areas of photonics, as well as SPIE Press Sr. Editor Dara Burrows for her help.

This book is dedicated to my mom, Albina.

Galina Nemova
September 2019

Brief History of Laser Cooling

Cooling (or **refrigeration**) is a physical process in which a substance is maintained at a temperature below that of its surroundings. It uses controlled heat removal and dates back thousands of years to when people first tried to preserve food. Ice and snow placed in holes in the ground or cold cellars were the only cooling techniques available. In the 19th century, scientists liquefied permanent gases and developed **cryogenics**.

In 1960, the first **laser** was built, revolutionizing cryogenics such that **laser cooling** (or **optical refrigeration**) and **trapping** were developed.

- In late 1950 and the early 1960s, **ion traps** were introduced.
- In 1962, Gurgen Askar'yan showed that intensity gradients could exert substantial forces on atoms due to the induced dipole moment.
- In 1968, Vladilen Letokhov proposed using the dipole force to trap atoms.
- In 1975, it was realized that atoms and ions can be laser cooled using a Doppler mechanism (**Doppler cooling**).
- In 1980, laser cooling with **single trapped ions** was achieved.
- In 1985, a laser-cooled atomic beam was trapped by a magnetic field. Soon after, **forced evaporation** was developed.
- In 1987, a much deeper trap, known as a **magneto-optical trap (MOT)**, was demonstrated.
- In 1988, it was realized that, using the multilevel character of atoms and the spatial variation of light polarization, **sub-Doppler (polarization gradient or Sisyphus) cooling** and a mean velocity as low as $3\text{--}5 v_{rec}$ (v_{rec} is the recoil velocity) can be achieved.
- Between 1988 and 1992, different sub-recoil schemes of laser cooling were proposed.
- In 1995, **Bose–Einstein condensate (BEC)** was realized in a dilute atomic vapor, and **laser cooling of solids** with **anti-Stokes fluorescence (ASF)** was achieved with a $\text{Yb}^{3+}:\text{ZBLAN}$ sample.

Laser Cooling and Trapping of Atoms and Ions

Laser cooling describes the cooling of a physical system (atoms, ions, or solids) upon interaction with laser light. In **gases**, thermal energy is primarily contained in translational degrees of freedom. In **solids**, thermal energy is contained in lattice vibrations (**phonons**). Although different approaches are required to cool atoms, ions, and solids, the general goal of cooling these systems is the same: to reduce the **kinetic energy** with laser light. Laser cooling and laser trapping are two different but closely related concepts of light–matter interaction.

Laser cooling is the *deceleration* of neutral atoms or ions by a velocity-dependent light force. It does not trap particles since they can diffuse out of the laser beam.

Laser trapping is the *confinement* of neutral atoms or ions within a small region by a spatially dependent light force. The **optical Earnshaw theorem** states that stable **trapping** *cannot* be achieved with radiation pressure force for particles with **scalar polarizabilities**. For example, for an atom with a $J_g = 0 \rightarrow J_e = 1$ transition, a ground state with total angular momentum quantum number J_g and an excited state with $J_e = J_g + 1$ are involved in the trapping process. An optical molasses alone cannot trap atoms. A static magnetic field can lead to nonscalar polarizabilities and, therefore, generate a stable radiation pressure trap.

For a magnetic field $\mathbf{B} \ll 1$ T (tesla), the depth of a **magnetic trap (MT)** is much less than 1 K (kelvin). Unlike a purely MT, a MOT, which uses the same principle, not only traps but also cools atoms, producing a trapped **optical molasses**. According to the Earnshaw theorem, ion traps cannot be constructed with purely electrostatic fields alone. This principle is applied in the design of **Paul** and **Penning traps**, which are widely used for trapping ions. Unlike traps for neutral atoms, which depend on the electronic state of the atom, the confining mechanism of ion traps does not rely on the internal structure of the ion. The electrical charge of ions allows them to be cooled and trapped with static electric and magnetic fields. The repulsive forces between ions make it difficult to realize BEC.

Methods of Laser Cooling

Laser cooling methods can be divided into three groups: Doppler laser cooling, sub-Doppler laser cooling, and sub-recoil laser cooling. The most widely used is **Doppler laser cooling**, which is based on the Doppler effect and can be applied to neutral atoms and ions. Although most atoms and ions have complicated hyperfine structures, in Doppler laser cooling, they are considered as simple two-level systems. The minimum temperature achievable is the **Doppler temperature** ($T_D \approx 100 \mu\text{K}$).

Sub-Doppler laser cooling (below T_D) is based on the hyperfine structures of atoms and two counter-propagating laser beams with orthogonal polarization, and can be realized with various laser-cooling techniques. For example, **Sisyphus cooling**, commonly used for neutral atoms, can serve as a bridge between Doppler cooling and ground-state laser cooling of ions. The lowest temperature that can be reached with Sisyphus cooling is the **recoil limit** ($T_{\text{Sis}} \approx 0.1\text{--}1 \mu\text{K}$). The recoil limit is reached when the thermal energy equals the energy of an atom with a momentum equal to the photon momentum.

Temperatures below the recoil limit can be reached with **sub-recoil laser cooling**, which includes techniques for neutral atoms and ions. The two most successful sub-recoil schemes for atoms are **velocity-selective coherent population trapping (VSCPT)**, where atoms with velocity $v \approx 0$ are decoupled from the light as a result of destructive interference between the two terms contributing to the excitation rate, and **Raman cooling**, which uses Raman scattering of laser light by the atoms. Raman cooling can be applied to both optical molasses (**free-space Raman cooling**) and to optical molasses where an optical lattice has been superimposed (**Raman sideband cooling**). **Resolved-sideband cooling** allows for cooling of strongly trapped atoms and ions (usually precooled with Doppler cooling) to the quantum ground state of their motion. Ground-state **electromagnetically induced transparency (EIT) cooling**, which uses EIT on two coupled dipole transitions, can be applied to trapped ions and atoms.

Potential and Kinetic Energy

Energy is a measure of a system's ability to do work. It exists in two forms:

Potential energy is the stored energy of position possessed by an object. For example, **gravitational potential energy** is the energy stored in an object as a result of its vertical position or height. **Elastic potential energy** is the energy stored in elastic materials as a result of their stretching or compressing.

Kinetic energy is the energy of motion and is associated with the velocity of a body. The kinetic energy E_{kin} of an object of mass m that is moving with velocity v can be calculated from the amount of work required to accelerate the body from rest to its current velocity:

$$E_{kin} = \frac{1}{2}mv^2$$

Kinetic energy can be transferred between objects and transformed into other kinds of energy.

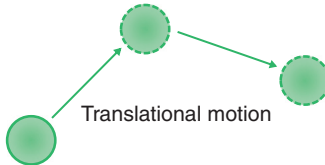
There are many forms of kinetic energy:

- **Rotational**—the energy due to rotational motion:

Rotational motion



- **Translational**—the energy due to motion from one location to another:



- **Vibrational**—the energy due to vibrational motion:

Vibrational motion



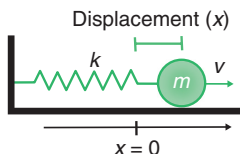
In the International System of Units (SI), the units of energy are given in **joules** (J).

Conservation Laws

The total energy of an isolated system remains constant over time (the **law of conservation of energy**). The energy can neither be created nor destroyed; it can only be transformed from one form to another.

For example, in a linear harmonic oscillator, the total energy E shifts back and forth between kinetic energy ($mv^2/2$) and potential energy ($kx^2/2$), where m is the mass, x is the displacement, v is the velocity, and k is the spring constant:

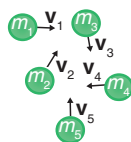
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$



The momentum \mathbf{p} of a body is the product of its mass m and its velocity vector \mathbf{v} :

$$\mathbf{p} = m\mathbf{v}$$

The total momentum of the system of particles is $\mathbf{P}_\Sigma = \sum_{i=1}^N m_i \mathbf{v}_i$.



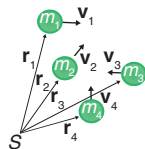
Conservation of linear momentum:

If no net external force acts on a system of particles, its total linear momentum \mathbf{P}_Σ remains constant.

The angular momentum \mathbf{L}_{cl} of a mass m at displacement \mathbf{r} and moving with velocity \mathbf{v} from origin S is defined as

$$\mathbf{L}_{cl} = \mathbf{r} \times m\mathbf{v} = \mathbf{r} \times \mathbf{p}$$

with respect to a reference frame with origin S .



The total angular momentum of a system of particles is

$$\mathbf{L}_\Sigma = \sum_{i=1}^N \mathbf{r}_i \times \mathbf{p}_i$$

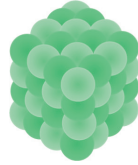
Conservation of angular momentum:

If the net moment due to external forces acting on a system of particles is zero, the total angular momentum \mathbf{L}_Σ of the system remains constant.

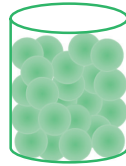
Gases, Liquids, Solids, and Plasma

Gases, liquids, solids, and plasma are all made up of atoms, molecules, and/or ions, but the behavior of these particles depends on the phase of the material.

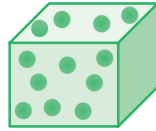
In **solids**, intermolecular forces are strongly attractive and bind particles together. Particles are tightly packed, usually in a regular pattern by strong interatomic and weak intermolecular bonds. In structural terms, all solids can be classified as crystals or amorphous solids. Particles in a solid vibrate but generally do not move from place to place.



Particles in a **liquid** are close together with no regular arrangement. They vibrate, move about, and slide past each other. Liquids fall somewhere between gases and solids. Some disordered materials (e.g., glass) have properties between those of liquids and solids.

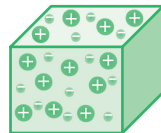


Particles in a **gas** are well separated with no regular arrangement. They vibrate, rotate, and move freely at high speeds, and interact with each other only when they collide.



- For a **monatomic gas**, the translational kinetic energy is also the total internal energy. Monatomic gases cannot have vibrational energy because a vibrational mode involves changing the distance or angle between atoms in a single molecule. Rotational kinetic energy can be ignored because the atoms are so small that the moment of inertia is negligible.
- For **gases made up of molecules** with more than one atom per molecule, the internal energy includes translational kinetic energy as well as energy associated with the bonds and the vibration of the molecules.

A medium in which all atoms are fully ionized is called a **plasma**. The number of free electrons in a plasma is approximately the same as the number of positive ions.

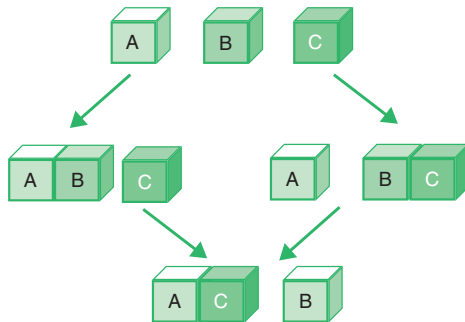


The Zeroth Law of Thermodynamics

The portion of the universe that is of interest in thermodynamics is called a **system**. A system may be a container filled with gas, a block of ice, a collection of objects, a human body, etc. The rest of the universe is called the **surroundings**. The surroundings is where we start to make observation on the system. The system is defined by its boundary:

- If matter can be added to or removed from the system, it is said to be an **open system**.
- A system with a boundary that is impervious to matter is called a **closed system**.
- A system with a boundary that allows it to remain unchanged regardless of any occurrence in the surroundings is called an **isolated system**.

The random motions of the atoms or molecules in a system A can be transferred to a system B via collisions in a conduction process called **thermally transferred energy**. Referring to the figure, A, B, and C are systems that can be brought into contact (top row). The **zeroth law of thermodynamics** states that if system A is in thermal equilibrium with system B (middle left), and system B is in thermal equilibrium with system C (middle right), then system C is in thermal equilibrium with system A (bottom row). The three systems have the same **temperature**.



The zeroth law of thermodynamics is the basis of the working principle of a **thermometer**, a device for measuring temperature.

Temperature and Thermometers

Any object with a **thermometric property**—a physical property that changes in a measurable way as the object is heated or cooled—can be used for building a thermometer.

Three conventions define a **temperature scale**:

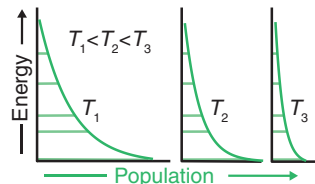
- selection of a suitable thermometric property,
- selection of a temperature dependence on this property, and
- selection of an appropriate number of calibration points.

The **Celsius** (°C) and the **Fahrenheit** (°F) temperature scales are empirical scales based on the use of fixed points. On the Celsius scale, 0 °C and 100 °C correspond to the freezing and boiling points of water, respectively. On the Fahrenheit scale, 0 °F is the lowest temperature attainable with a mixture of salt, ice, and water. 100 °F is the temperature of the human body. The **Kelvin** (K) temperature scale is based on absolute zero and is the official **International Practical Temperature Scale (IPTS)** unit of temperature.

Absolute zero is the lowest theoretically attainable temperature at which particles stop moving. The kinetic energy of atoms and molecules at absolute zero is minimal.

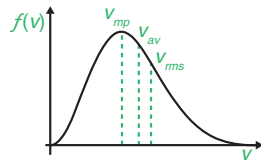
In quantum mechanics, atoms have only discrete values of energy. Temperature is a parameter that expresses the equilibrium distribution of the molecules (or the atoms) of a system over their energy levels. At a given temperature T , a collection of atoms includes some atoms in their lowest (ground) state, some in the first excited state, etc. This distribution is called the **Boltzmann** (or **Gibbs**) **distribution**.

The ratio of the population $f(E_2)$ of the energy state E_2 to the population $f(E_1)$ of the energy state E_1 is $f(E_2)/f(E_1) = \exp[(E_1 - E_2)/k_B T]$, where k_B is the **Boltzmann constant**. If $T = 0$ K, only the lowest state is occupied; if $T = \infty$, all states are equally populated.



Maxwell–Boltzmann Distribution

Gas molecules (ideal gas) in a container have different speeds that correspond to certain kinetic energies. The **Boltzmann distribution** can be used to express the distribution of molecules over their possible translational energy states or their distribution of speeds, and to relate their distribution of speeds to the temperature. The resulting expression is the **Maxwell–Boltzmann distribution** of speeds:



$$f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2k_B T} \right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}}$$

where T is the temperature of the gas, and m and v are the particle mass and speed, respectively.

The **most probable speed** v_{mp} (or **peak speed**) corresponds to the maximum of the speed distribution. The largest number of particles travel at this speed. Indeed,

$$\frac{df(v)}{dv} = 0 \Rightarrow v_{mp} = \sqrt{\frac{2k_B T}{m}}$$

The **average speed** v_{av} of a molecule is slightly higher than the most probable speed because the distribution is not symmetrical. The longer “tail” on the right side of the Maxwell–Boltzmann distribution pulls the average speed slightly to the right of the peak in the graph:

$$v_{av} = \int_0^{\infty} v f(v) dv = \sqrt{\frac{8k_B T}{\pi m}}$$

The root-mean-square (rms) speed is the square root of the mean (average) of the squares of the velocities. We use the **rms speed** v_{rms} when dealing with the kinetic energy of molecules:

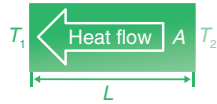
$$v_{rms} = \sqrt{\int_0^{\infty} v^2 f(v) dv} = \sqrt{\frac{3k_B T}{m}}$$

In gases made of molecules, rotational energy can be converted to translational energy, and vice versa, due to molecular collisions that result in vibrations along the lines joining the atoms in the molecules.

Thermal Physics

Thermal equilibrium occurs when all masses within a closed system have the same temperature because the total energy is equally divided among all particles of the combined system. It can be achieved by one or more heat transfer processes: conduction, convection, and radiation.

Heat conduction is a microscopic heat transfer process. More-energetic particles, including molecules, atoms, and electrons, transfer their kinetic and potential energy



to less-energetic particles through collisions. If one end of a solid bar is kept at a fixed temperature T_1 and the other end is kept at T_2 ($T_2 > T_1$), the rate of heat energy flow through the bar is $\dot{Q} = C(T_2 - T_1)$, where $C = \kappa A/L$ is thermal conductance (a property of the *object*), κ is the thermal conductivity (a property of the *material*), A is the surface area, and L is the length.

A hot solid object suspended in heated fluid rises, and the fluid is replaced by cooler fluid. Heat energy leaves the object, thus cooling it (**natural convection**). This process can be accelerated with mechanical assistance, e.g., a fan (**forced convection**). In this case, $\dot{Q} = hA(T_2 - T_1)$, where h is the **heat transfer coefficient**, and A is the surface area.



Any body at a temperature above absolute zero emits **radiant energy**, i.e., electromagnetic (EM) waves in the **thermal range** ($0.1\text{--}100\ \mu\text{m}$), according to the **Stefan–Boltzmann law**, $\dot{Q} = \epsilon\sigma AT^4$, where σ is the Stefan–Boltzmann constant, and ϵ is the surface emissivity. This law indicates the ability of a body to emit or absorb radiation, which varies between 0 and 1, and depends on the nature and the temperature of the surface. The **emissivity** depends on the wavelength of the **radiation** being emitted or absorbed. The most efficient radiators are surfaces with $\epsilon \rightarrow 1$, which absorb radiation with equal efficiency. If $\epsilon = 1$, all radiation is absorbed (a **black-body**). The radiation emitted by a blackbody surface does not depend on its composition but only on its temperature, and is called the **Planck spectrum**. A body in vacuum can lose heat by radiation only.