

## 11.1 Optimization Approaches

There are four techniques to employ optimization of optical structures with optical performance constraints:

- Level 1 is characterized by manual iteration to improve the predicted performance of a design. In this approach, a finite element analysis is performed to find the structural deflections. The FE results are processed in a postprocessor to write surface deformations in a format readable by optical analysis software, as described earlier in this text. The optical analysis software is then used to compute optical performance. Intuition and experience are important in this process to recognize how the design should be modified to improve performance.
- Level 2 is characterized by the use of equations of optical performance within the FE model. These equations can be written for optical performance quantities at the single-surface level, such as surface RMS error after bias, tilt, and power have been removed, or at the system level, such as RMS wavefront error or line-of-sight jitter. The internal optimizer in FE software can then optimize the optical design directly without the need for manual iterations.
- Level 3 is characterized by calculation of optical performance through an external subroutine linked to the FE software for use by the FE program's optimizer. This approach may be used to perform optimization using design performance metrics that cannot be computed by the equations used in Level 2. One such example is the design optimization of an adaptively controlled mirror in order to minimize the corrected surface figure.
- Level 4 is characterized by combining the capabilities of CAD, FE, and optical analysis within a single optimization program. This level of implementation allows coupled design variation of the optical prescription and the mechanical design. There has been some notable progress in this approach, but it is not yet commonplace.

This chapter includes a brief overview of optimization theory and its terminology. However, the main emphasis is on the application of optimization tools to optomechanical systems. In the design optimization of a typical optical structure the predicted quantities relating to performance of the system are referred to as design responses. Example design response types are shown in Table 11.2. Generally, only one of these design responses may be specified to be minimized or maximized by the optimizer and is referred to as the objective. All other design responses may have performance limits applied to them consistent with the requirements of the design. The applications of such limits in the optimizer are referred to as design constraints. In order to define how the optimizer is allowed to modify the design, several types of parameters and the

**Table 11.2** Typical design response quantities used in the optomechanical design optimization.

<b>TYPICAL DESIGN RESPONSE QUANTITIES:</b>	
<1>	<b>Structural: System weight, center-of-gravity, mass-moment-of-inertia</b>
<2>	<b>Structural: Stress, buckling, natural frequency, dynamic response</b>
<3>	<b>Optical: Image motion, jitter, MTF</b>
<4>	<b>Optical: Surface RMS error</b>
<5>	<b>Optical: System wavefront error</b>

**Table 11.3** Typical design-optimization problem statement.

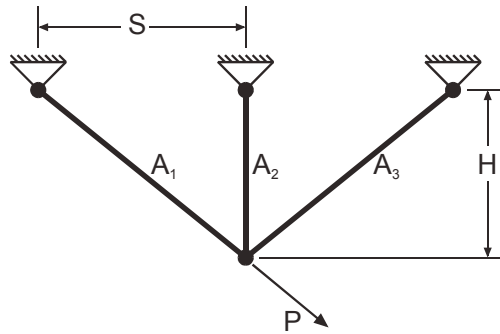
<b>DEFINITIONS:</b>		
$X$	= vector of design variables, such as sizing, shape, material	
$R$	= vector of design responses, typically nonlinear functions of $X$	
$F$	= objective = a design response to minimize or maximize	
$g$	= design constraint on a response as either an upper or lower bound	
	$R \leq R^U \Rightarrow g = (R - R^U) / R^U \leq 0$ (11.1)	
<b>MATHEMATICAL DESIGN PROBLEM STATEMENT:</b>		
Minimize	$F(X)$	
subject to	$g \leq 0$	behavior constraints
and	$X^L \leq X \leq X^U$	side constraints (11.2)

manner with which they relate to the structural design may be specified. These parameters are referred to as design variables. The design variables often have specific allowable limits and are referred to as side constraints. Side constraints differ from design constraints in that side constraints are applied to design variables, whereas design constraints are applied to design responses. Table 11.3 further illustrates the definitions of a design optimization problem and shows a complete design-optimization problem statement.

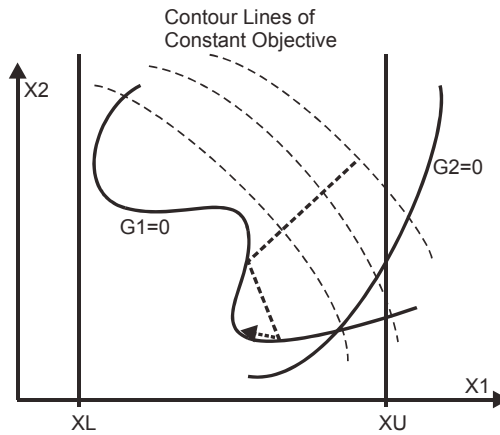
Current technology allows for structural optimization using optical performance constraints (Section 11.3) or multidisciplinary thermal-structural-optical optimization (Section 11.4). This chapter does not address some other problems that could broadly fall under optomechanical design, such as optical beam path length optimization<sup>1</sup> in which optimization is used to solve a difficult geometry problem.

## 11.2 Optimization Theory

In this text, optimum design refers to the application of nonlinear programming techniques to find the best solution of the mathematical statement of the design problem.



**Figure 11.1** Three-bar truss with truss member of areas  $A_1$ ,  $A_2$ , and  $A_3$  as labeled.



**Figure 11.2** Two-variable design space.

If the design goal is to maximize the objective  $F$ , the problem can be stated in standard form by minimizing  $-F$ . If a response is limited by an equality constraint, it may be treated as two inequality constraints:

$$h = 0 \Rightarrow h \leq 0 \text{ and } h \geq 0. \quad (11.3)$$

Fig. 11.1 shows a simple three-bar truss to be optimized with cross-sectional area sizing variables  $A_1$ ,  $A_2$ , and  $A_3$ , and shape variables  $S$  and  $H$ . The objective is to minimize the weight of the structure while satisfying performance constraints on displacement and stress, and obeying side constraints on size and shape.

The design space is an  $N$ -dimensional space with an axis for each of the  $N$  design variables, which is impossible to visualize if  $N > 3$ . A two-variable design space is depicted in Fig. 11.2. In most problems, the constraints are generally nonlinear functions of  $X$  and are often found numerically, which makes them expensive and difficult to plot, even in a 2D space. In the five-variable truss example, the stress and displacement are found via FEA, and all responses are nonlinear in  $S$  and  $H$ .

There is a variety of NLP techniques available<sup>2</sup> that move through the design space in a sequential manner. The most efficient techniques are gradient-based, requiring first derivatives (sensitivities) of the response quantities with respect to the design variables ( $dR/dX$ ).

A common approach is to use finite differences to calculate sensitivities. Let  $X_0$  represent a starting design point:

$$X_0 = (A_1, \dots, A_j, \dots, A_n), \quad (11.4)$$

which is evaluated via FEA:

$$K_0 U_0 = P_0 \Rightarrow U_0. \quad (11.5)$$

The derivative of displacement with respect to design variable  $A_j$  is found by perturbing the design:

$$X_j = (A_1, \dots, A_j + \Delta A_j, \dots, A_n). \quad (11.6)$$

Then, re-evaluating with FEA,

$$K_j U_j = P_j \Rightarrow U_j, \quad (11.7)$$

and computing a finite difference derivative:

$$U_j' = dU/dX_j = (U_j - U_0)/\Delta A_j. \quad (11.8)$$

This is a very general technique, but quite expensive computationally.

A more efficient technique uses implicit derivatives of the initial equation [Eq. (11.5)]:

$$K_0 U' + K' U_0 = P'. \quad (11.9)$$

The derivative  $U'$  can be solved from

$$K_0 U' = P' - K' U_0 = P^*, \quad (11.10)$$

which is the equivalent computational cost of an additional load case  $P^*$  in the original solution. Note that  $K'$  and  $P'$  are relatively computationally inexpensive to calculate in most cases. For the example truss problem,

$$k = AE/L \Rightarrow k' = dk/dA = E/L. \quad (11.11)$$

For external forces,  $P'$  is 0. For a gravity body force,

$$P = AL\rho g / 2 \Rightarrow P' = dP / dA = L\rho g / 2. \quad (11.12)$$

Most other design responses can then be found from  $U'$  by the chain rule. For example, the stress sensitivity in the truss is found from

$$d\sigma / dX = (d\sigma / dU)U' \Rightarrow d\sigma / dU = E / L. \quad (11.13)$$

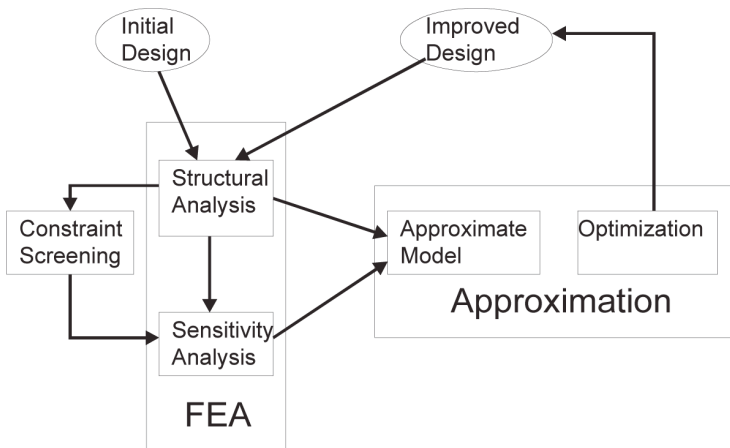
Typical design optimizations require more than 100 design cycles to optimize. For large models, the computational time for 100 analyses is prohibitive. A significant efficiency can be gained by using the design sensitivities and approximation theory<sup>2</sup> to create a design response surface. The steps in this approach are

- <1> give a design  $X_q$  at design cycle  $q$ ,
- <2> run a full FE analysis along with design sensitivity,
- <3> create approximate problem (response surface) via Taylor series:

$$g^* = g(X_q) + g'(X_q)/(X - X_q), \quad (11.14)$$

- <4> optimize the approximate problem very quickly to get  $X_{q+1}$ ,
- <5> check convergence before looping back to Step 1.

In this approach (shown in Fig. 11.3), Step 4 requires hundreds of computationally inexpensive optimizations, while the computationally expensive FEA in Step 2 is typically 10–20 analyses.



**Figure 11.3** Optimization flow using approximation theory.