Chapter 14
Neurophysical Model by Barten and Its Development

According to the Barten model, the perceived foveal image is corrupted by internal noise caused by statistical fluctuations, both in the number of photons that trigger photoreceptor excitation and in the signal transport from photoreceptors to the brain. While the former effect is easily identified with the photon shot noise intrinsic to any luminous flux, the latter effect is related to the process of subdivision and recombination of photocurrents generated by each individual cone. These photocurrents create a parallel stream of information that is conveyed through the complex mesh of neural cells (horizontal, bipolar, amacrine, and ganglion) that form the retina–brain connection.[1] Thus, fluctuations in the electrical/biochemical transport through such parallel pathways result in small differences in the image elements arriving at the brain (neural noise).

The overall effect of noise establishes a threshold level below which an image cannot be perceived or distinguished without a high probability of error. The process can be compared to the common experience of image degradation induced by fog in open air, relative to a clear atmosphere, or by flicker in a television image. The disturbance (fog or flicker) plays the role of noise externally added to an otherwise sharp scene, and limits the resolving capability of our visual perception.

The Barten model is based on the following simple relationship:

$$m(\psi) \cdot |MTF_T(\psi)| = SNR \cdot N_T(\psi), \tag{14.1}$$

which states that, at the perception threshold, modulation $m(\psi)$ at angular spatial frequency ψ of the object being observed, filtered by the total MTF (MTF_T) of the eye–brain system, must overcome the total noise level $N_T(\psi)$ at the same frequency by a suitable factor, represented by the signal-to-noise ratio (SNR). In general, the object imaged by the eye can be any pattern of structural and chromatic complexity; however, it is mathematically convenient to refer to black-and-white sinusoidal luminance gratings of variable spatial frequency that form the base for spectral decomposition of any 2D object. In this case, modulation $m(\psi)$—also termed Michelson contrast $c(\psi)$—is defined as

$$m(\psi) = c(\psi) = \frac{L(\psi)_{max} - L(\psi)_{min}}{L(\psi)_{max} + L(\psi)_{min}}, \tag{14.2}$$

which is the amplitude of the sinusoidal variation of luminance $\left(\frac{L(\psi)_{max} - L(\psi)_{min}}{2}\right)$ divided by average luminance $\left(\frac{L(\psi)_{max} + L(\psi)_{min}}{2}\right)$.

In Eq. (14.1), MTF_T appears in modulus form (even if unnecessary by definition) to stress that possible sign reversal must not be included, as all of the other parameters in the equation are positively defined quantities.

By choosing sinusoidal gratings, modulation $m(\psi)$ of Eq. (14.1) takes the meaning of threshold modulation function $TMF(\psi)$, that is, the smallest modulation of a sinusoidal luminance grating that can be recognized by an observer (a physical parameter that is easily quantified in human eyes through the usual measurement of CS at some fixed spatial frequencies).

Figure 14.1 shows a representation of the effect of noise at various values of SNR (∞, 5, 3, and 2) on the visibility of sinusoidal gratings for a single spatial frequency and various contrast levels (c = 1; 0.3; and 0.1). In each frame of Fig. 14.1, the scale is such that the entire frame linear size corresponds to 100 arcmin, and the pixel size (visible in noisy patterns) is equivalent to the cone spacing in the fovea center (about 0.5 arcmin). The spatial frequency represented in Fig. 14.1 thus corresponds to 12 cpd.

By adopting convenient expressions for $MTF_T(\psi)$ and $N_T(\psi)$ (illustrated in the following), Barten was able to solve Eq. (14.1) for $m(\psi) \equiv TMF(\psi)$, and to derive an adequate formula for $CSF(\psi)$, the reciprocal function of $TMF(\psi)$:

$$CSF(\psi) \equiv \frac{1}{TMF(\psi)} = \frac{|MTF_T(\psi)|}{SNR \cdot N_T(\psi)}, \tag{14.3}$$

where all functions are implicitly assumed to also depend on wavelength λ. By means of Eq. (14.3), Barten successfully fit a number of experimental data series of CS taken from the literature.

14.1 Total MTF

In the Barten model outlined in Eq. (14.3), MTF_T refers to the overall spatial filtering experienced by the visual signal in the complete transport chain through the eye, along the optical nerve up to the brain. It is made up of three contributions: optical component MTF_O, pertinent to the refractive media of the eye; retinal component MTF_R, describing the

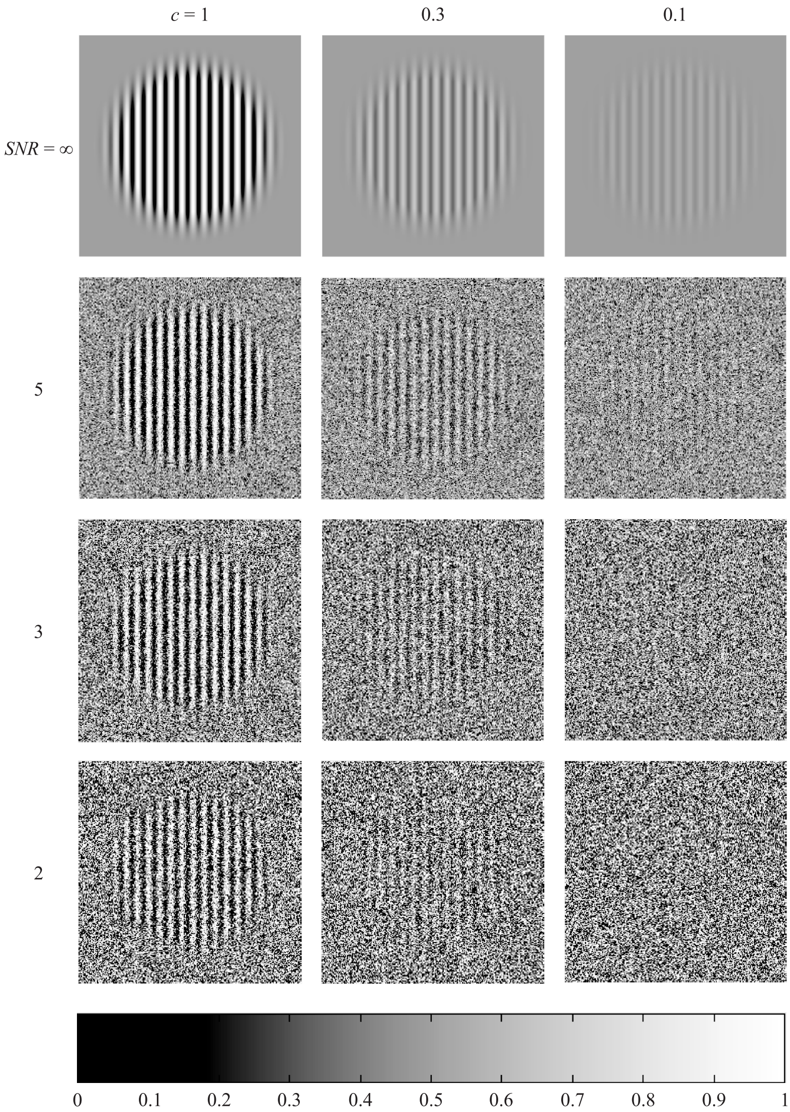

Figure 14.1 Visualization of the effect of noise on the detectability of sinusoidal gratings, for three values of contrast ($c = 1$; 0.3; and 0.1), and four levels of SNR ($SNR \to \infty$; 5; 3; and 2). Assuming that the pixel size in each frame corresponds to the cone spacing in the fovea (≈0.4 arcmin), the sinusoidal bars are 1.5 deg long and spatial frequency is 12 cpd.

discrete sampling structure of the fovea; and neural component MTF_N, characterizing the filtering action suffered by the visual signal in its transport along neurons from photoreceptors to the brain. Thus, MTF_T is given by

$$MTF_T(\psi) = MTF_O(\psi) \cdot MTF_R(\psi) \cdot MTF_N(\psi). \tag{14.4}$$

The retinal and neural components are assumed to be wavelength independent.

14.1.1 Optical MTF

In his analysis of CS data, Barten adopted a simple heuristic formula to represent MTF_O of the ocular refractive media: $MTF_O(\psi) = e^{-2\pi^2[\sigma_0^2+(C\cdot D_{ent})^2]\psi^2}$, given by a Gaussian function with a line width inversely dependent on pupil size, to account for spherical and chromatic aberrations. In this expression, σ_0 and C are constant parameters, and D_{ent} is the size of the entrance pupil. Despite the simplicity of the model, the visual agreement of the data fits he obtained was remarkably good. MTF line width σ_0 was left as a fitting parameter, to be freely adjusted for maximum adherence of model to data. However, a better solution is represented by use of expressions for MTF_O calculated with a physical optics approach. In the present context, $MTF_O(\psi)$ is given by the numerical results obtained for the CAGE eye model and described in Part IB. The resulting shape of MTF_O then depends only on the size of entrance pupil D_{ent}, the spectral composition of the visual stimulus, and amount of defocus.

14.1.2 Retinal MTF

The sampling action provided by foveal cones on the luminance signal falling onto the retina alters the signal by transforming its spatial distribution from continuous to step-wise, thus introducing a limit to visual resolution due to the cone mosaic.

Barten did not specify a formula for MTF_R, but instead included it into his heuristic MTF_O. In the present case, a suitable expression for the sampling function can be easily devised. Assume for simplicity that the incident foveal irradiance is actually the line spread function $LSF(\theta)$, with θ being the angular coordinate, so that its Fourier transform directly results in MTF_O. Suppose further that foveal cones are regularly distributed in a 2D arrangement, each one having angular width W and spacing S (center-to-center angular distance). Finally, assume that in the sampling process, foveal irradiance is spatially integrated across the cone aperture and is lost in the dead zone between adjacent photoreceptors. Mathematically, the sampled LSF can be written as[2]

$$LSF_R(\theta) = \Sigma(\theta) \cdot \sum_k \delta_k(\theta - kS), \tag{14.5}$$

where $\delta(\theta)$ is Dirac delta function, and the sampling function $\Sigma(\theta)$ is given by

$$\begin{aligned}\Sigma(\theta) &= \frac{1}{W}\int_{-\infty}^{\infty} LSF(u) \cdot \mathrm{rect}\left(\frac{\theta - u}{W/2}\right) du \\ &= \frac{1}{W}\mathrm{conv}\left\{LSF(\theta), \mathrm{rect}\left(\frac{\theta}{W/2}\right)\right\}.\end{aligned} \tag{14.6}$$

In this formula, $\mathrm{rect}\left(\frac{\theta}{W/2}\right)$ defines the finite unitary impulse function of width W (see Appendix A), and conv represents the convolution operator. In this way, $\Sigma(\theta)$ is continuously defined on the θ axis, while $LSF_R(\theta)$ is defined only at the center of each photoreceptor (or *pixel*).

The corresponding $MTF_R(\theta)$ after sampling can thus be evaluated as[3]

$$MTF_R(\psi) = FT\left\{LSF_R(\theta)\right\} = \sum_k MTF\left(\psi - \frac{k}{S}\right) \cdot \left|\mathrm{sinc}\left[\pi W\left(\psi - \frac{k}{S}\right)\right]\right|. \tag{14.7}$$

As a consequence of sampling, the original double-sided spectrum $MTF(\psi)$ is attenuated by the sinc $(\cdot)$ function (FT of the active sampling window of size W), and replicated at frequency intervals k/S (where $1/S$ is the sampling or Nyquist angular frequency). Given that the domain of $MTF(\psi)$ is $[-\psi_0, \psi_0]$, with $\psi_0 = \frac{D_{ex}}{\lambda}$, ($D_{ex}$ being the size of the exit pupil), then aliasing, that is, the overlap of adjacent spectral orders, occurs whenever $2\psi_0 > \frac{1}{S}$ that is, $D_{ex} > \frac{\lambda}{2S}$.

Numerical values for S and W can be obtained from the literature. The classical work by Curcio and Allen[4] provides a mean value for the maximum density of foveal cones of 210,000 mm^{-2} (range: 120,000 to 324,000), confirmed by a recent measurement of 199,200 mm^{-2} (Ref. 5). By adopting density value $n_C = 200{,}000\ \mathrm{mm}^{-2}$—with a retinal magnification factor given by $\gamma = 4.96\ \mu\mathrm{m/arcmin}$ (see Section 10.1)—the cone spacing for a square cell arrangement becomes $S = \frac{10^3}{\gamma\sqrt{n_C}} \cong$ 0.45 arcmin. For a pattern of hexagonal cells, the separation between adjacent cell rows is given by $S = \frac{10^3}{\gamma}\sqrt{\frac{\sqrt{3}}{2n_C}} \cong 0.42$ arcmin. Assuming 30% for the fraction of dead space between adjacent cones yields $W =$ 0.35 arcmin. By comparing Eqs. (14.4) and (14.7), the retinal MTF_R of Eq. (14.4) can be set equal (for order $k = 0$) to $MTF_R(\psi) = |\mathrm{sinc}(\pi W\psi)|$.